Applied Mathematics Letters 53 (2016) 85–91

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Exponential stability for a structure with interfacial slip and frictional damping

C.A. Raposo

Department of Mathematics - Federal University of São João del-Rei, UFSJ Praça Frei Orlando, 170, São João del-Rei-MG, Brazil

ARTICLE INFO

Article history:
Received 14 July 2015
Received in revised form 7 October 2015
Accepted 8 October 2015
Available online 23 October 2015

Keywords:
Exponential stability
Laminated beam
Interfacial slip
Energy method

ABSTRACT

In this work we prove the exponential stability for a laminated beam consisting of two identical layers of uniform density, which is a system closely related to the Timoshenko beam theory, taking into account that an adhesive of small thickness is bonding the two layers and produce the interfacial slip. It is assumed that the thickness of the adhesive bonding the two layers is small enough so that the contribution of its mass to the kinetic energy of the entire beam may be ignored.

1. Introduction

There are few papers that deal with systems of interfacial slip, we cite [1–5]. In [1], Hansen and Spies derived the mathematical model (1)–(3) for two-layered beams with structural damping due to the interfacial slip

$$\rho u_{tt} + G(\psi - u_x)_x = 0, \quad x \in (0,1),\ t \geq 0, \tag{1}$$

$$I_\rho(3S_{tt} - \psi_{tt}) - G(\psi - u_x) - D(3S_{xx} - \psi_{xx}) = 0, \quad x \in (0,1),\ t \geq 0, \tag{2}$$

$$3I_\rho S_{tt} + 3G(\psi - u_x) + 4\delta_0 S + 4\gamma_0 S_t - 3DS_{xx} = 0, \quad x \in (0,1),\ t \geq 0, \tag{3}$$

where $u(x,t)$ denotes the transverse displacement, $\psi(x,t)$ represents the rotation angle, and $S(x,t)$ is proportional to the amount of slip along the interface at time t and longitudinal spatial variable x. The coefficients $\rho, G, I_\rho, D, \delta_0, \gamma_0$ are the density, the shear stiffness, mass moment of inertia, flexural rigidity, adhesive stiffness, and adhesive damping of the beams. Eq. (3) describes the dynamics of the slip.

E-mail address: raposo@ufsj.edu.br.

http://dx.doi.org/10.1016/j.aml.2015.10.005
0893-9659/

Now, we consider $s(x,t) = 3S(x,t)$ and $\rho_1 = \rho$, $\rho_2 = I_\rho$, $k = G$, $b = D$, $3\delta = \delta_0$, $3\gamma = \gamma_0$, then we deduce from (1)–(3) the following system

$$\rho_1 u_{tt} + k(\psi - u_x)_x = 0, \tag{4}$$

$$\rho_2 (s-\psi)_{tt} - b(s-\psi)_{xx} - k(\psi - u_x) = 0, \tag{5}$$

$$\rho_2 s_{tt} - b s_{xx} + 3k(\psi - u_x) + 4\delta s + 4\gamma s_t = 0. \tag{6}$$

Combining (5) and (6) we have

$$\rho_1 u_{tt} + k(\psi - u_x)_x = 0, \tag{7}$$

$$\rho_2 \psi_{tt} - b\psi_{xx} + 4k(\psi - u_x) + \delta s + \beta s_t = 0. \tag{8}$$

Putting $s = 0$ in (8) we obtain the following system

$$\rho_1 u_{tt} + k(\psi - u_x)_x = 0,$$
$$\rho_2 \psi_{tt} - b\psi_{xx} + 4k(\psi - u_x) = 0,$$

that is a conservative system closely related with the Timoshenko's beam.

In [3] was proved that the frictional damping created by the interfacial slip alone is not enough to stabilize the system (1)–(3) exponentially to its equilibrium state. In recent paper, see [5], was showed that for viscoelastic material there isn't need for any kind of internal or boundary control to stabilize exponentially the system of laminated beams with interfacial slip while in [6] was proved that for Timoshenko's beams with viscoelastic damping, the exponential stability holds, if and only if, the velocities of propagations are equal, that is, $\rho_1 b = \rho_2 k$. In this direction, the viscoelastic damping may have different influence when acts in the stabilization of Timoshenko's system and when is effective on the structure with interfacial slip. Now for frictional damping, if we introduce the damping αu_t on the transverse displacement and $\beta\psi_t$ on the rotation angle in the Timoshenko's system, we obtain the dissipative system

$$\rho_1 u_{tt} + k(\psi - u_x)_x + \alpha u_t = 0, \tag{9}$$

$$\rho_2 \psi_{tt} - b\psi_{xx} + k(\psi - u_x) + \beta\psi_t = 0, \tag{10}$$

that is exponentially stable, see for example [7] and reference therein.

Motivated by the exponential stability of (9)–(10), we intent to investigate the action of the frictional damping in the system (1)–(3), more precise, we consider the following system

$$\rho_1 u_{tt} + k(\psi - u_x)_x + \alpha u_t = 0, \tag{11}$$

$$\rho_2 (s-\psi)_{tt} - b(s-\psi)_{xx} - k(\psi - u_x) + \beta(s-\psi)_t = 0, \tag{12}$$

$$\rho_2 s_{tt} - b s_{xx} + 3k(\psi - u_x) + 4\delta s + 4\gamma s_t = 0, \tag{13}$$

where $(x,t) \in (0,L) \times (0,\infty)$ with boundary conditions

$$u(0,t) = \psi(0,t) = s(0,t) = 0, \tag{14}$$

$$s_x(L,t) = \psi_x(L,t) = 0, \qquad u_x(L,t) = \psi(L,t), \tag{15}$$

and initial data

$$(u(x,0), \psi(x,0), s(x,0)) = (u_0(x), \psi_0(x), s_0(x)) \in \mathcal{H} = [H_0^1(0,L)]^3, \tag{16}$$

$$(u_t(x,0), \psi_t(x,0), s_t(x,0)) = (u_1(x), \psi_1(x), s_1(x)) \in \mathcal{V} = [L^2(0,L)]^3. \tag{17}$$

For the system (1)–(3) in [2] was proved the existence and uniqueness of solution in the class

$$(u,\psi,s) \in C\left([0,T] : \mathcal{H}\right) \cap C^1\left([0,T] : \mathcal{V}\right).$$