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Bilinear equations and new multi-soliton solution for the modified Camassa–Holm equation

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ABSTRACT

The bilinear equations, which directly derive the modified Camassa–Holm equation, are given from the reduction of the extended KP hierarchy with negative flow. As a byproduct, the general N-soliton solution in the parametric form to the modified Camassa–Holm equation is obtained from the tau functions of the extended KP hierarchy in the form of Gram-type determinant.

1. Introduction

In this paper, we focus on the modified Camassa–Holm (mCH) equation

$$m_t + [m(u^2 - u_x^2)]_x = 0, \quad m = u - u_{xx}, \tag{1}$$

which originally appeared as a new integrable system in the works of Fokas [1] and Fuchssteiner [2] as well as Olver and Rosenau [3]. As mentioned by Fokas, this equation arises in the theory of nonlinear water waves, which describes qualitative properties in the fully nonlinear regime with nonlinear dispersion. Both of the Camassa–Holm equation and the modified Camassa–Holm equation describe the breakdown of nonlinear waves and support a notable variety of non-smooth soliton-like solutions. As well known, applying the tri-Hamiltonian duality to the bi-Hamiltonian representation of the KdV equation leads to the well-studied Camassa–Holm equation [1–6]. Similarly, if one applies tri-Hamiltonian duality to the mKdV equation, then the corresponding dual system is exactly the mCH equation (1) with cubic nonlinearity [3,7].

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The mCH equation has attracted much attention and has been studied extensively in recent years. From the geometric view, the mCH equation is shown to arise from an intrinsic (arc-length preserving) invariant planar curve flow in Euclidean geometry [8]. It is worth mentioning that the mCH equation has new features due to its higher-order nonlinearity, including wave breaking and multi-peakon dynamics, which differ from the known properties of the CH equation [9,10]. Specifically, under the vanishing boundary condition, the mCH equation exhibits the usual peakons [8] whereas under the nonvanishing boundary condition, it supports smooth bright soliton solutions [11]. Matsuno constructed the N-soliton solution to the mCH equation (1) through Hirota's bilinear method [12]. However, the mCH equation is not derived directly from the provided bilinear equations. In the present paper, by combining the extended KP hierarchy reduction method and Hirota's bilinear method, we intend to find the bilinear equations which can derive the mCH equation directly. As a byproduct, the N-soliton solution to the mCH equation (1) in the Gram-type determinant is presented.

The outline of the paper is as follows. In Section 2, a set of two bilinear equations is given, which is shown to derive the mCH equation through dependent variable and hodograph (reciprocal) transformations. In Section 3, it is shown that the above two bilinear equations can be obtained from the reduction technique of the extended KP hierarchy. As a byproduct, the N-soliton solutions are obtained in Section 4. Section 5 is devoted to a brief summary and discussions.

2. Bilinearization of the mCH equation

In this section, the bilinear form of the mCH equation is proposed by means of dependent variable transformation and the hodograph transformation.

Theorem 2.1. *The following two bilinear equations*

$$(2D_\tau D_y^2 + 2D_\tau D_y - 4D_y)g \cdot f = 0, \tag{2}$$

$$(D_y^2 + D_y)g \cdot f = 0, \tag{3}$$

derive the mCH equation (1) through the hodograph transformation

$$x = y + \tau + 2\ln\frac{g}{f}, \tag{4}$$

$$t = \tau, \tag{5}$$

and the dependent variable transformation

$$u = 1 - (\ln fg)_{y\tau}, \tag{6}$$

where D_x is the Hirota D-operator, which is defined as

$$D_x^n f \cdot g = \left(\frac{\partial}{\partial x} - \frac{\partial}{\partial y}\right)^n f(x)g(y)\,|_{y=x}\,.$$

Proof. Firstly, we give two bilinear identities:

$$\begin{aligned}\frac{D_\tau D_y^2 g\cdot f}{g\cdot f} &= \left(\ln\frac{g}{f}\right)_{yy\tau} + 2(\ln fg)_{y\tau}\left(\ln\frac{g}{f}\right)_y \\ &\quad + (\ln gf)_{yy}\left(\ln\frac{g}{f}\right)_\tau + \left(\ln\frac{g}{f}\right)_y^2\left(\ln\frac{g}{f}\right)_\tau,\end{aligned} \tag{7}$$

$$\frac{D_\tau D_y g\cdot f}{g\cdot f} = (\ln gf)_{y\tau} + \left(\ln\frac{g}{f}\right)_y\left(\ln\frac{g}{f}\right)_\tau. \tag{8}$$