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Global existence of solutions to a nonlinear anomalous diffusion system

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ABSTRACT

A nonlinear system with different anomalous diffusion terms is considered. The existence of global positive solutions is proved.

1. Introduction

In this paper, we consider the following system of nonlinear fractional in space reaction–diffusion equations

$$\begin{cases} u_t(x,t) + t^a(-\Delta)^{\alpha}\, u(x,t) \;=\; A(t)\, v^r(x,t)\, u^s(x,t), & x \in \mathbb{R}^N,\ t>0, \\ v_t(x,t) + t^b(-\Delta)^{\beta}\, v(x,t) \;=\; B(t)\, v^p(x,t)\, u^q(x,t), & x \in \mathbb{R}^N,\ t>0, \end{cases} \tag{1.1}$$

for $u > 0, v > 0$, equipped with the initial conditions

$$u(x,0) = u_0(x), \quad v(x,0) = v_0(x),\ x \in \mathbb{R}^N, \tag{1.2}$$

where the initial data

$$u_0, v_0 \in C_{LB}(\mathbb{R}^N) := \left\{\phi \in C(\mathbb{R}^N) : \phi \ge c > 0\right\},$$

are given functions, $a > 0$, $b > 0$, $s < 0$, $p < 0$, $r < 1 - p$ and $q < 1 - s$. The functions A and B are such that $A(t) \ge c_0 t^k$ and $B(t) \ge c_1 t^l$ where $c_0 > 0$, $c_1 > 0$, $k > 0$ and $l > 0$.

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Here the nonlocal operator $(-\Delta)^{\mu}$, $0 < \mu \leq 1(\mu = \alpha, \beta)$ stands for anomalous diffusion [1] and is defined, for any function u in the Schwartz space, by the Fourier transform pair $\mathcal{F}$ and $\mathcal{F}^{-1}$

$$(-\Delta)^{\mu}u(x) = \mathcal{F}^{-1}\big(|\xi|^{2\mu}\mathcal{F}u(\xi)\big)(x);$$

it has the Reisz representation

$$(-\Delta)^{\mu}u(x) = C_{N,\mu}PV\int_{\mathbb{R}^N}\frac{u(x)-u(y)}{|x-y|^{N+2\mu}}\,dy,$$

where $C_{N,\mu}$ is a normalizing constant depending only on N and μ [2], such that $\lim_{\mu\to 1^-}(-\Delta)^{\mu}\phi = -\Delta\phi$. We will show that system (1.1)–(1.2) admits globally bounded solutions relying on a comparison argument.

Heat equations and systems have received a great attention concerning global existence and blow-up of solutions; they are well documented and we only refer to the important books [3–6]. However, the work on parabolic fractional differential equations is scarce and it started with the paper of Nagasawa and Sirao [7] who used a probabilistic treatment of blowing-up solutions to equations with fractional powers of the Laplacian of the form

$$u_t + (-\Delta)^{\alpha}u = c(x)\,|u|^p\,,$$

for a certain positive function $c(x)$. Sugitani [8] treated the same equation with $c(x) = 1$, while Kobayashi [9] discussed a more general equation. Concerning systems of fractional in space differential equations, we can mention the papers [10,11] and [12]. Kirane et al. [13] investigated a further extension to fractional in time and space systems.

The system (1.1) when it pops up with positive exponents for the nonlinear terms appears in combustion theory [3] while the cases for $s = p = 0$, $r = -2$ and $q = -2$ appear in the mathematical analysis of micro-electromechanical systems (MEMS) and have important applications such as accelerometers for airbag deployment in cars, inject printer heads, etc., for instance, see [14,15] and [16].

Our system has both positive and negative exponents even though the negative exponents do not pop up as in the MEMS system.

2. Preliminaries

First of all, we recall some fundamental facts.

Let $S_\alpha(t)$ be the semi-group associated with the heat equation

$$u_t + (-\Delta)^{\alpha}u = 0, \quad 0 < \alpha \leq 1,\ t > 0,\ x \in \mathbb{R}^N.$$

It is known that $S_\alpha(t)$ defined by

$$S_\alpha(t)(x) = \frac{1}{(2\pi)^{\frac{N}{2}}}\int_{\mathbb{R}^N} e^{ix\xi - t|\xi|^{\alpha}}\,d\xi$$

satisfies the following properties:

- $S_\alpha(t) \in L^\infty(\mathbb{R}^N) \cap L^1(\mathbb{R}^N)$;
- $S_\alpha(t) \geq 0$ and $\int_{\mathbb{R}^N} S_\alpha(t)dx = 1$, $x \in \mathbb{R}^N$, $t > 0$,

and the following estimates:

- $\|S_\alpha(t) * u_0\|_p \leq \|u_0\|_p$, $u_0 \in L^p(\mathbb{R}^N)$, $1 \leq p \leq \infty$, $t > 0$;
- $\|S_\alpha(t) * u_0\|_q \leq ct^{-\frac{N}{\alpha}(\frac{1}{p}-\frac{1}{q})}\|u_0\|_p$;
- $\|\nabla S_\alpha(t)\|_q \leq ct^{-\frac{N}{\alpha}(1-\frac{1}{q})-\frac{1}{\alpha}}$ for any $u_0 \in L^p(\mathbb{R}^N), 1 \leq p < q \leq \infty$, $t > 0$.