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Approximate solution of the problem of scattering of surface water waves by a partially immersed rigid plane vertical barrier

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ABSTRACT

The problem of scattering of two dimensional surface water waves by a partially immersed rigid plane vertical barrier in deep water is re-examined. The associated mixed boundary value problem is shown to give rise to an integral equation of the first kind. Two direct approximate methods of solution are developed and utilized to determine approximate solutions of the integral equation involved. The all important physical quantity, called the Reflection Coefficient, is evaluated numerically, by the use of the approximate solution of the integral equation. The numerical results, obtained in the present work, are found to be in an excellent agreement with the known results, obtained earlier by Ursell (1947), by the use of the closed form analytical solution of the integral equation, giving rise to rather complicated expressions involving Bessel functions.

1. Introduction

Weakly singular integral equations of the first kind arise frequently in the research problems of various fields of mathematical physics [1–3]. Several analytical and numerical methods have been devised to determine solutions to this type of integral equations [4–7]. Here we propose two numerical methods to solve a weakly singular integral equation connected with a special mixed boundary value problem arising in the study of water wave scattering by a thin partially immersed barrier in deep water. This problem was first solved analytically by Ursell [8] by reducing it to a first kind singular integral equation of the form

$$\int_a^\infty f(u)\Big[Kln\Big|\frac{y+u}{y-u}\Big| - \frac{1}{y+u} - \frac{1}{y-u}\Big]\mathrm{d}u = 0, \quad a < y < \infty \tag{1}$$

where K is a constant.

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That the integral is interpreted in the sense of Cauchy principal value, is represented by '$-$' sign on the integral. The unknown function $f(y)$ which represents the fluid velocity along the line below the thin plate behaves as $f(t) \sim O\left(|t-a|^{-\frac{1}{2}}\right)$ as $t \to a$. The integral Eq. (1) was solved by Ursell [8] analytically and the reflection and the transmission coefficients were determined in terms of Bessel functions.

This problem was also solved by Williams [9] through a weakly singular integral equation approach. Variants of Williams' method were developed by Chakrabarti [10,11]. If the water depth is finite, it is not possible to obtain the exact solution and researchers employed different methods to obtain numerical estimates for the physical quantities. Some of the notable research work in this direction are found in Parsons and Martin [12], Abul-Azm [13], Porter and Evans [14]. As the number of problems with exact solutions is quite limited, with the advent of high speed computers, researchers are involved in developing algorithms with faster rate of convergence. In our present work, we make an effort to solve Ursell's problem by reducing it to a non-homogeneous first kind integral equation and solving it by polynomial approximations of the unknown function together with collocation at suitable points. In order to achieve this, we first reduce the governing boundary value problem in terms of a first kind non-homogeneous integral equation for determining the velocity function across the vertical line below the thin barrier. Using the behaviour of the velocity at the end point of the plate and at infinity, the unknown velocity is represented as a product of a known elementary function and an unknown smooth function. Two different ways of approximating the smooth function give rise to two different methods. In the first approach the function is expressed to an unknown polynomial of degree N. This polynomial is substituted into the integral equation and the free variable is collocated at the finite number of points by the zeros of Chebyshev polynomial of the second kind. The second approach consists of approximating the unknown function by an unknown series of Chebyshev polynomials. Both the procedures yield systems of linear algebraic equations when collocated at finite number of points. The linear systems are solved to determine the discrete numerical values of the unknown function. Using these values the reflection coefficient is determined numerically. Graphical representation of the reflection curve reveals that the results exactly coincide with those of Ursell [8]. Thus the current analysis presents two simple techniques for solving a first kind integral equation in which the unknown function has prescribed end behaviour. As a test, the methods have been employed to solve a problem whose exact solution is well established in the literature. The methods are quite general and can be employed to problems for which analytical solutions do not exist.

2. Mathematical formulation

Making the usual assumptions of linearized theory of water waves, the boundary value problem for the governing velocity potential $\phi(x, y)$ is described by the following equations (cf. Mandal and Chakrabarti [15])

$$\nabla^2\phi = 0 \quad \text{in the fluid region,} \tag{2}$$

along with the free surface boundary condition

$$K\phi + \frac{\partial\phi}{\partial y} = 0 \quad \text{on } y = 0, \tag{3}$$

the condition on the barrier

$$\frac{\partial\phi}{\partial x} = 0, \tag{4}$$

and the boundary condition on the bottom

$$\nabla\phi \to 0 \quad \text{as } y \to \infty. \tag{5}$$