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H_∞ control for a class of discrete-time singular systems via dynamic feedback controller[☆]

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ABSTRACT

This paper deals with the H_∞ control problem for a class of discrete-time singular systems. A dynamic feedback control scheme is considered in this paper, while few results in the existing literature employed it to study the H_∞ control problem for the discrete-time singular systems. By using the Lyapunov functional method and the linear matrix inequality (LMI) approach, a sufficient condition for the existence of a dynamic feedback controller that guarantees the resulting closed-loop system to be admissible with given H_∞ performance index is derived. Finally, a numerical example is provided to show the effectiveness of the proposed method.

1. Introduction

Singular systems, which are also called generalized state-space systems, semi-state systems or descriptor systems, can describe physical systems better than regular ones [1–4] since a singular system model involves not only differential equations (or difference equations) but also algebraic equations. The applications of singular systems can be found in many practical systems, such as power systems, circuits systems and economic systems [5]. It is noted that the study of the singular systems is often much more difficult than that of regular ones because it usually requires to consider the regularity and impulse elimination (for continuous-time singular systems) or causality (for discrete-time singular systems) simultaneously, and the two need not be considered in the regular ones. During the past several decades, singular systems have been studied extensively and a number of important results have been presented in the literature, see, e.g., [5–10].

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In real physical systems, one is often faced with external disturbances that may cause instability and poor performance. Therefore, it should be taken into account the effect of the disturbances in the study of many practical systems. Since one of the attractive features of the H_∞ control is to keep the systems less sensitive to the disturbances, a great deal of attention has been dedicated to the H_∞ control problem and the highly-related problems of H_∞ control such as H_∞ filtering and H_∞ model reduction, see, e.g., [9–17]. There also have been some results concerning the H_∞ control problem for the discrete-time singular systems [18–21]. In [18], the design method was presented in the form of a set of semi-definite and nonlinear matrix inequalities that are difficult to calculate. [19] studied the H_∞ control problem for uncertain discrete-time singular systems and the results were presented in terms of LMI. [20] and [21] employed a novel bounded real lemma to further study the H_∞ control problem for discrete-time singular systems and uncertain discrete-time singular systems, respectively.

However, the designed controller in the literature [18–21] is based on the static state-feedback control scheme. It is well known that the dynamic feedback controller can provide more flexibility compared to the static controller and the apparent advantage of the dynamic feedback controller is that it provides more free parameters for selection. To the best of our knowledge, few results in the existing literature investigated the H_∞ control problem for the discrete-time singular systems via the dynamic feedback controller. This motivates the present work of this paper.

This paper deals with the H_∞ control problem for a class of discrete-time singular systems by using the dynamic feedback controller. Based on the Lyapunov functional method and the LMI approach, a sufficient condition for the existence of a dynamic feedback controller that guarantees the resulting closed-loop system to be admissible with given H_∞ performance index is presented. Finally, we give a numerical example to show the effectiveness of the presented approach.

Notations: $\|\cdot\|$ stands for the Euclidean norm for a vector. R^n denotes the n-dimensional Euclidean space. $R^{m\times n}$ is the set of all $m \times n$ real matrices. For a real symmetric matrix X, $X > 0 (X \geq 0)$ means that X is positive definite (semi-positive definite). I is the identity matrix of appropriate dimensions. The symbol "$*$" denotes the symmetric elements in a symmetric matrix. $\lambda_{\max}(\cdot)$ means the largest eigenvalue of a matrix. The superscript "T" stands for the transpose of a matrix or a vector. The space of square summable infinite sequence is denoted as $\ell_2[0,\infty)$.

2. Preliminaries

Consider the discrete-time singular system as follows:

$$Ex(k+1) = Ax(k) + B_w\omega(k) + Bu(k), \qquad y(k) = Cx(k) + D\omega(k), \qquad x(0) = x_0, \tag{1}$$

where $x(k) \in R^n$ is the state vector, $u(k) \in R^m$ is the input vector, $\omega(k) \in R^q$ is the disturbance vector and belongs to $\ell_2[0,\infty)$ and $y(k) \in R^p$ is the output vector. $E \in R^{n\times n}$ is a known real constant matrix and assumed to be singular and satisfy $0 < \operatorname{rank}(E) = r < n$. A, B, B_w, C and D are known real constant matrices with appropriate dimensions.

In this paper, we consider the following dynamic feedback controller:

$$E\zeta(k+1) = \overline{A}\zeta(k) + \overline{B}x(k), \qquad u(k) = \overline{C}\zeta(k), \qquad \zeta(0) = 0, \tag{2}$$

where $\zeta(k) \in R^n$ is the controller state. $\overline{A}$, $\overline{B}$ and $\overline{C}$ are with appropriate dimensions to be determined later.

Combining (1) and (2), we obtain the following dynamics:

$$E_c\xi(k+1) = A_c\xi(k) + B_{wc}\omega(k), \qquad y(k) = C_c\xi(k) + D\omega(k), \tag{3}$$

where $E_c = \begin{bmatrix} E & 0 \\ 0 & E \end{bmatrix}$, $A_c = \begin{bmatrix} A & B\overline{C} \\ \overline{B} & \overline{A} \end{bmatrix}$, $B_{wc} = \begin{bmatrix} B_w \\ 0 \end{bmatrix}$, $C_c = \begin{bmatrix} C & 0 \end{bmatrix}$ and $\xi(k) = \begin{bmatrix} x(k) \\ \zeta(k) \end{bmatrix}$.