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Energy decay for a von Karman equation with time-varying delay

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ABSTRACT

We consider a von Karman equation with time-varying delay of the form

$$u_{tt}(x,t) + \Delta^2 u(x,t) + a_0 h_1(u_t(x,t)) + a_1 h_2(u_t(x,t-\tau(t))) = [u(x,t), F(u(x,t))].$$

By introducing suitable energy and Lyapunov functionals, we establish decay estimates for the energy, which depends on the behavior of h_1.

1. Introduction

We investigate a decay result of the energy for a von Karman equation with time-varying delay

$$u_{tt} + \Delta^2 u + a_0 h_1(u_t(x,t)) + a_1 h_2(u_t(x,t-\tau(t))) = [u, F(u)] \quad \text{in } \Omega \times (0,\infty), \tag{1.1}$$

$$\Delta^2 F(u) = -[u,u] \quad \text{in } \Omega \times (0,\infty), \tag{1.2}$$

$$u = \Delta u = 0, \qquad F(u) = \frac{\partial F(u)}{\partial \nu} = 0 \quad \text{on } \partial\Omega \times (0,\infty), \tag{1.3}$$

$$u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x) \quad \text{in } \Omega, \tag{1.4}$$

$$u_t(x,t) = f_0(x,t) \quad \text{in } \Omega \times [-\tau(0), 0), \tag{1.5}$$

where $\Omega \subset \mathbb{R}^2$ is a bounded domain with sufficiently smooth boundary $\partial\Omega, \nu = (\nu_1, \nu_2)$ is the outward unit normal vector on $\partial\Omega$, $x = (x_1, x_2) \in \Omega$, $a_0 > 0$, $a_1 \neq 0$ is a real number, $\tau(t) > 0$ represents time-varying delay, h_1, h_2 are given functions, (u_0, u_1, f_0) belong to a suitable function space, and von Karman bracket $[u,\phi]$ is given by $[u,\phi] \equiv u_{x_1x_1}\phi_{x_2x_2} + u_{x_2x_2}\phi_{x_1x_1} - 2u_{x_1x_2}\phi_{x_1x_2}$.

When $a_1 = 0$ and $a_0 = 0$ in (1.1), Favini et al. [1] proved global existence, uniqueness and regularity of solutions for the equation with nonlinear boundary dissipation, moreover they showed the uniqueness of

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weak solutions by proving sharp regularity results of the Airy stress function, Park and Park [2] considered the existence of strong solutions and uniform decay rates for the equation with boundary memory condition. When $a_1 = 0$ and $h_1(u_t)$ is replaced by $b(x)u_t$ in (1.1), Horn and Lasiecka [3] investigated energy decay rates of weak solutions for the equation with nonlinear boundary dissipation. For related works of von Karman equations with dissipative effects, we refer [4–7] and references therein. The aim of this work is to prove a general decay result for the von Karman equation with time-varying delay appearing in the control term in (1.1). Considering the time-varying delay term $a_1 h_2(u_t(x, t - \tau(t)))$, the problem is different from existing literature. Time delays arise in many applications depending not only on the present state but also on some past occurrences. And the presence of delay may be a source of instability (see e.g. [8,9]). Thus, the control of partial differential equations with time delay effects has become an active area of research (see [9–11] and references therein). As regards a wave equation with delay of the form

$$u_{tt}(x,t) - \Delta u(x,t) + a_0\sigma(t)h_1(u_t(x,t)) + a_1\sigma(t)h_2(u_t(x,t-\tau(t))) = 0. \tag{1.6}$$

Nicaise and Pignotti [9] proved that the energy of the problem decays exponentially when $\sigma(t) = 1, a_0, a_1 > 0$, $\tau(t) = \tau(\text{constant})$, and $h_1(s) = h_2(s) = s$. Benaissa et al. [12] studied problem (1.6) when $a_0, a_1 > 0$, σ, h_1, h_2 satisfy some conditions. They proved existence and general decay results by using multiplier technique. Inspired by these results, we prove a general decay result for problem (1.1)–(1.5) by dropping the restriction $a_1 > 0$ and establishing suitable Lyapunov functionals.

2. Preliminaries and main results

We denote $V_0 = \{u \in H^3(\Omega) |\ u = \Delta u = 0 \text{ on } \partial\Omega\}, V = \{u \in H^4(\Omega) |\ u = \Delta u = 0 \text{ on } \partial\Omega\}$, and $(u, v) = \int_\Omega u(x)v(x)dx$. For a Banach space X, $\|\cdot\|_X$ denotes the norm of X. For simplicity, we denote $\|\cdot\|_{L^2(\Omega)}$ by $\|\cdot\|$. Let λ_0 and λ be the best constants in the Poincaré type inequalities $\lambda_0\|u\|^2 \le \|\nabla u\|^2$ and $\lambda\|u\|^2 \le \|\Delta u\|^2$, respectively. In this paper, we shall omit x and t in all functions of x and t if there is no ambiguity. We state the assumptions for the problem (1.1)–(1.5).

(A1) Similarly to [13], $h_1 : \mathbb{R} \to \mathbb{R}$ is a non-decreasing function of the class $C(\mathbb{R})$. There exist positive constants r, c_1, c_2 and a convex increasing function $H_1 : \mathbb{R}_+ \to \mathbb{R}_+$ of class $C^1(\mathbb{R}_+) \cap C^2((0,\infty))$ satisfying $H_1(0) = 0$, and H_1 is linear on $(0, r]$ or ($H_1'(0) = 0$ and $H_1''(t) > 0$ on $(0, r]$) such that

$$c_1|s| \le |h_1(s)| \le c_2|s| \quad \text{for } |s| \ge r, \tag{2.1}$$

$$s^2 + h_1^2(s) \le H_1^{-1}(sh_1(s)) \quad \text{for } |s| \le r. \tag{2.2}$$

(A2) $h_2 : \mathbb{R} \to \mathbb{R}$ is an odd non-decreasing function of the class $C^1(\mathbb{R})$ such that there exist positive constants c_i, $i = 3, 4, 5$, satisfying

$$|h_2'(s)| \le c_3 \quad \text{and} \quad c_4 s h_2(s) \le H_2(s) \le c_5 s h_1(s) \quad \text{for } s \in \mathbb{R}, \tag{2.3}$$

where $H_2(s) = \int_0^s h_2(t)dt$.

(A3) As in [11], $\tau \in W^{2,\infty}([0, T])$ for $T > 0$ and there exist positive constants τ_0, τ_1, and d satisfying

$$0 < \tau_0 \le \tau(t) \le \tau_1 \quad \text{and} \quad \tau'(t) \le d < 1 \quad \text{for all } t > 0. \tag{2.4}$$

(A4) The weight of dissipation and the delay satisfy

$$0 < |a_1| < \frac{c_4(1-d)}{c_5(1-c_4 d)} a_0. \tag{2.5}$$