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Exponential stability of neutral stochastic delay differential equations with Markovian switching

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ABSTRACT

In this paper, by using the Lyapunov stability theory, Dynkin's formula, matrix theory, neutral differential equations theory and stochastic analysis techniques, we study the pth moment exponential stability for neutral stochastic delay differential equations (NSDDEs) with Markovian switching, $p \geq 1$. Some new conditions are derived to obtain the pth moment exponential stability of the trivial solution. At last, an example is presented to show the effectiveness of the proposed results.

1. Introduction

NSDDEs provide an excellent mathematical modeling framework for numerous applications in natural science and technology. Since they are important in the development of theory and practice, they have attracted many researchers' interests. The main interest has been focused on the stability criteria for NSDDEs, see for instance [1–12].

NSDDEs with Markovian switching which is an important class of hybrid dynamical systems have been successfully applied in practice, such as in traffic control, switching power converters, neural networks, and so on. The research to the stability of NSDDEs with Markovian switching, we refer the readers to [13–22] and the references therein.

In the now available results, for example, in [1,3], the authors mainly investigated exponential stability in the mean square of neutral stochastic functional differential equations. In [23], the authors chiefly discussed exponential stability in the mean square of Markovian jump neural networks with impulse control and time varying delays. In [21], the authors principally studied stochastic global exponential stability in the

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mean square for neutral-type impulsive neural networks with mixed time-delays and Markovian jumping parameters. To the best of our knowledge, except for [13], the published papers are not considered on the pth moment exponential stability of NSDDEs with Markovian switching. In [13], the authors mainly used the Lyapunov stability analysis techniques to obtain the result. Of course, these Lyapunov stability analysis techniques are very good methods, but at times the Lyapunov functional is not very easy to be discovered in real systems. Thus, in our paper, we consider special forms of Lyapunov–Krasovskii functional inspired by [21,23–25] to gain simpler sufficient conditions on the pth moment exponential stability of NSDDEs with Markovian switching.

In reference to the existing consequences in the literature, we make the following contributions. (i) We use the Lyapunov–Krasovskii functional to investigate the pth moment exponential stability of NSDDEs with Markovian switching. (ii) We overcome the difficulty bringing out by the Markovian switching in discussing the pth moment exponential stability of NSDDEs with Markovian switching.

The rest of this paper is organized as follows: In Section 2, some preliminaries are offered. In Section 3, one theorem on the pth moment exponential stability of NSDDEs with Markovian switching is established by using Lyapunov–Krasovskii functional. In Section 4, an example is presented to show the application of the proposed results.

Notation. The following notations are needed in this paper. Let $(\Omega, \mathcal{F}, \{\mathcal{F}_t\}_{t\geq 0}, P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e. it is right continuous and $\mathcal{F}_0$ contains all P-null sets), and $E[\cdot]$ stand for the correspondent expectation operator with respect to the given probability measure P. Let $\omega(t) = (\omega_1(t), \dots, \omega_m(t))^T$ be an m-dimensional Brownian motion defined on a complete probability space with a natural filtration. Let $|\cdot|$ denotes the Euclidean norm in $\mathbb{R}^d$. If A is a vector or matrix, its transpose is denoted by A^T. Trace $(\cdot)$ denotes the trace of the corresponding matrix and I denotes the identity matrix with compatible dimensions. If A is a matrix, its trace norm is denoted by $|A| = \sqrt{trace(A^TA)}$ while its operator norm is denoted by $\|A\| = \sup\{|Ax| : |x| = 1\}$. If A is a symmetric matrix, denote by $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ its largest and smallest eigenvalue, respectively. For square matrices A_1 and A_2, the notation $A_1 > (\geq, <, \leq) A_2$ denotes $A_1 - A_2$ is a positive-definite (positive-semi-definite, negative, negative-semi-definite) matrix.

2. Model description and preliminaries

In this section, we will introduce the model and some preliminary lemmas.

Let $\tau > 0$ and $C([-\tau, 0]; \mathbb{R}^d)$ denote the family of continuous function ψ from $[-\tau, 0]$ to $\mathbb{R}^d$, with the norm $\|\psi\| = \sup_{-\tau\leq\theta\leq 0} |\psi(\theta)|$.

For $p > 0$, denote $C^p_{\mathcal{F}_t}([-\tau, 0]; \mathbb{R}^d)$ by the family of all $\mathcal{F}_t$-measurable $C([-\tau, 0]; \mathbb{R}^d)$-valued random variables ψ such that $\sup_{-\tau\leq\theta\leq 0} E|\psi(\theta)|^p < \infty$.

Let $r(t), t \geq 0$ be a right-continuous Markov chain on the probability space taking values in the finite state space $S = 1, 2, \dots, N$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta) = j | r(t) = i\} = \begin{cases} \gamma_{ij}\Delta + o(\Delta) & \text{if } i \neq j, \\ 1 + \gamma_{ij}\Delta + o(\Delta) & \text{if } i = j, \end{cases}$$

where $\Delta > 0$. Here $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$ while

$$\gamma_{ii} = -\sum_{j\neq i} \gamma_{ij}.$$

We assume that the Markov chain $r(\cdot)$ is independent of the Brownian motion $\omega(\cdot)$. It is known that almost every sample path of $r(t)$ is a right-continuous step function with a finite number of simple jumps in any finite subinterval of $\mathbb{R}_+ (:= [0, \infty))$.