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Mild solutions of local non-Lipschitz stochastic evolution equations with jumps

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ABSTRACT

By estimating the coefficients functions in the stochastic energy equality, the existence and uniqueness of mild solutions to stochastic evolution equations (SEEs) under local non-Lipschitz condition proposed by Taniguchi with jumps are proved here. The results of Taniguchi (2009) are generalized and improved as a special case of our theory. It should be pointed that the proof for SEEs with jumps is certainly not a straightforward generalization of that for SEEs without jumps and some new techniques are developed to cope with the difficulties due to the Poisson random measures.

1. Introduction

In this work, we consider stochastic evolution equations (SEEs) with jumps in the following forms:

$$dX(t) = [A(t, X(t)) + f(t, X_t)]dt + g(t, X_t)dW(t) + \int_U k(t, X_t, y)\tilde{N}(dt, dy), \quad t > 0 \tag{1}$$

where $A(t, \cdot) : [0, T] \times V \to V^*$ is a linear or nonlinear bounded operator and the initial value $X(\theta) = \varphi(\theta) \in L^2(\Omega, D([-\delta, 0]; H)), \theta \in [-\delta, 0]$, the functions $f : [0, T] \times D_\delta(H) \to H$ and $g : [0, T] \times D_\delta(H) \to L_2^0(K, H)$ are progressively measurable, and the functions $h : [0, T] \times D_\delta(H) \times (F - \{0\}) \to H$ are predictably measurable.

SEEs which are widely used to model all kinds of behaviors in various fields of science such as mechanical engineering, control theory and economics have been extensively investigated by many authors [1,2]. Fruitful results have been achieved, such as existence, uniqueness, or stability of solutions to these SEEs and other quantitative and qualitative properties of them [3–5].

Specially, many scholars pay much attention to the existence, uniqueness to SEEs under much weaker sufficient conditions [6,7]. Unlike the global Lipschitz condition, the so-called non-Lipschitz condition is a

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much weaker sufficient condition with wider range of potential applications. This condition was investigated by many scholars. Yamada [8] and Xu [9] studied the solutions to stochastic differential equations (SDEs) under Yamada type non-Lipschitz condition. Taniguchi [10] generalized the Yamada type non-Lipschitz condition and studied the existence and uniqueness of solutions to SDEs under Taniguchi type non-Lipschitz condition which includes the special result of [8]. Pardoux [6] has studied the energy solutions which satisfy the energy equality to the non-Lipschitz SEEs. Cao et al. [7] considered the existence and uniqueness of solutions to non-Lipschitz SEEs driven by Poisson jumps.

Similar to study the case of local Lipschitz condition, the SEEs only with local non-Lipschitz condition proposed by Taniguchi [10] have recently attracted considerable attention. Taniguchi [11] discussed the existence, uniqueness to local non-Lipschitz SEEs driven by Wiener process. Ren [12] considered the mild solutions of neutral semilinear stochastic functional dynamic systems with local non-Lipschitz coefficients driven by Wiener process.

However, to the best of our knowledge, SEEs with jumps have not been studied sufficiently until now, even though they may be more appropriate to model the stochastic disturbances. In this paper, we will make the first attempt to study the SEEs with jumps for the case where the coefficients functions satisfy the local non-Lipschitz condition and the results of Taniguchi [11] are generalized and improved as a special case of our theory.

2. Preliminaries

Let V and H be separable Hilbert spaces with the norm $\|\cdot\|$ and $\|\cdot\|_H$, respectively, such that

$$V \subset H \equiv H^* \subset V^*,$$

where V is a dense subspace of H and the injections are continuous. We denote by $\langle \cdot, \cdot \rangle$ the duality between V and V^* and by $(\cdot, \cdot)$ the inner product in H.

Let $(\Omega, \mathcal{F}, P)$ be a complete probability space on which an increasing and right continuous family $(\mathcal{F})_{t\geq 0}$ and $\mathcal{F}_0$ contains all P-null sets of $\mathcal{F}$. Let $\delta > 0$ and $D_\delta(H) := D([-\delta, 0]; H)$ denote the family of all right-continuous functions with left-hand limits φ from $[-\delta, 0]$ to H with the notation $\|\varphi\|_{D_\delta} := \sup_{-\delta\leq s\leq 0} \|\varphi(s)\|_H$. Let $L^2(\Omega, D([-\delta, 0]; H))$ denote the space of all square integral, progressive measurable functions from $D([-\delta, 0]$ into H. Given $\delta > 0$ and $T > 0$, we denote by $I^2(-\delta, T; V)$ the space of V-valued process $(X(t))_{t\in[-\delta,T]}$ which is $\mathcal{F}$ measurable and satisfies $\int_{-\delta}^{T} E\|X(t)\|^2 dt < +\infty$.

Let $L(K, H)$ denote the space of all bounded linear operators from K to H. Let $\varrho_n(t)(n = 1, 2, 3, \ldots)$ be a sequence of real valued one-dimensional standard Brownian motions mutually independent on $(\Omega, \mathcal{F}, P)$. Set $W(t) := \sum_{n=1}^{\infty} \varrho_n(t)\sqrt{Q}e_n$, where $e_n(n = 1, 2, 3...)$ is complete orthonormal basis in K, $Q \in L(K, K)$ is nonnegative self-adjoint operator.

Furthermore, $L_2^0(K, H)$ denotes the space of all $\xi \in L(K, H)$ such that $\xi\sqrt{Q}$ is a Hilbert–Schmidt operator and so $tr(\xi Q\xi^*) < \infty$. The norm is given by $\|\xi\|^2_{L_2^0} = tr(\xi Q\xi^*)$. Then, ξ called the Q-Hilbert–Schmidt operator from K to H. We note that if $Q = I$, then $L_2^0(K, H)$ implies $L_2(K, H)$.

Let F be a separable Hilbert space and let $B_\sigma(F)$ denote the Borel $\sigma-$algebra of F. Let $p(t), t \geq 0$ be a stationary $\mathcal{F}_t$- adapted and F-valued Poisson point process. The counting random measure N_p defined by $N_p((0, t] \times \mathbb{Z}) := \sum_{0<s\leq t} I_{\mathbb{Z}}(p(s))$, for any $\mathbb{Z} \in B_\sigma(F)$, is the Poisson random measure associated to the Poisson point process p. Then we defined the compensated Poisson measure $\tilde{N}$ associated to the Poisson point process p by $\tilde{N}(dtdy) := N_p(dtdy) - v(dy)dt$, where v is a σ-finite measure and is called the Lévy measure.