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Solving the general Sylvester discrete-time periodic matrix equations via the gradient based iterative method

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ABSTRACT

The present work proposes a gradient based iterative method to find the solutions of the general Sylvester discrete-time periodic matrix equations

$$\sum_{j=1}^{m}\Big(A_{ij}X_iB_{ij}+C_{ij}X_{i+1}D_{ij}+E_{ij}Y_iF_{ij}+G_{ij}Y_{i+1}H_{ij}\Big)=M_i,$$
$$i=1,2,\ldots.$$

It is proven that the proposed iterative method can obtain the solutions of the periodic matrix equations for any initial matrices. Finally a numerical example is included to demonstrate the validity and applicability of the iterative method.

1. Introduction

This article considers the general Sylvester discrete-time periodic (GSDTP) matrix equations

$$\sum_{j=1}^{m}\Big(A_{ij}X_iB_{ij}+C_{ij}X_{i+1}D_{ij}+E_{ij}Y_iF_{ij}+G_{ij}Y_{i+1}H_{ij}\Big)=M_i,\quad i=1,2,\ldots,\tag{1.1}$$

where the coefficient matrices $A_{ij},C_{ij},E_{ij},G_{ij}\in\mathbb{R}^{p\times n}$, $B_{ij},D_{ij},F_{ij},H_{ij}\in\mathbb{R}^{n\times q}$, $M_i\in\mathbb{R}^{p\times q}$, and the solutions $X_i,Y_i\in\mathbb{R}^{n\times n}$ are periodic with period ω, i.e., $A_{i+\omega,j}=A_{i,j}$, $B_{i+\omega,j}=B_{i,j}$, $C_{i+\omega,j}=C_{i,j}$, $D_{i+\omega,j}=D_{i,j}$, $E_{i+\omega,j}=E_{i,j}$, $F_{i+\omega,j}=F_{i,j}$, $G_{i+\omega,j}=G_{i,j}$, $H_{i+\omega,j}=H_{i,j}$, $M_{i+\omega}=M_i$, $X_{i+\omega}=X_i$ and $Y_{i+\omega}=Y_i$ for $j=1,2,\ldots,m$ and $i=1,2,\ldots$. Solving the periodic matrix equations is of interest in linear periodic system theory, especially in the areas of optimal control, prediction and stability [1–7]. For example, the discrete-time periodic Lyapunov (DPL) matrix equations

$$A_kX_kA_k^T-X_{k+1}=-B_kB_k^T,\tag{1.2}$$

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and

$$A_k^T X_{k+1} A_k - X_k = -Q_k, \tag{1.3}$$

appear in the semi-global stabilization problem of discrete-time linear periodic (DLP) systems

$$x(k+1) = A_k x(k) + B_k u_k,$$

with the periodic matrices [3]. The GSDTP matrix equations (1.1) are general and contain various linear discrete-time periodic matrix equations such as the DPL matrix equations. So far some numerical methods had been developed to solve the periodic matrix equations. In [8], the Bartels–Stewart and Hessenberg–Schur algorithms were extended for periodic Lyapunov and Sylvester equations. Varga introduced efficient numerically reliable algorithms based on the periodic Schur decomposition for the solution of periodic Lyapunov matrix equations [9]. In [10], Kressner proposed new variants of the squared Smith iteration and Krylov subspace based methods for the approximate solution of discrete-time periodic Lyapunov equations. Recently Benner et al. discussed the numerical solution of large-scale sparse projected periodic discrete-time Lyapunov equations in lifted form which arise in model reduction of periodic descriptor systems. [11].

In this article, a gradient based iterative method is proposed to solve the GSDTP matrix equations (1.1). The solutions of the GSDTP matrix equations have not been dealt with yet.

The outline of this article is as follows. In Section 2, first we introduce a gradient based iterative method for solving (1.1) and second it is shown that the introduced iterative method converges to the solutions for any initial matrices. We report a numerical example to show the effectiveness of the introduced iterative method in Section 3.

2. Main results

In this section, first we give the necessary and sufficient conditions for the existence of solutions of the GSDTP matrix equations (1.1). Then a gradient based iterative method is presented to solve (1.1). At the end of this section, we show that presented method converges to the exact solutions for any initial matrices.

We can equivalently transform the GSDTP matrix equations (1.1) to the following general Sylvester matrix equation

$$\sum_{j=1}^{m} \Big(\mathcal{A}_j \mathcal{X} \mathcal{B}_j + \mathcal{C}_j \mathcal{X} \mathcal{D}_j + \mathcal{E}_j \mathcal{Y} \mathcal{F}_j + \mathcal{G}_j \mathcal{Y} \mathcal{H}_j \Big) = \mathcal{M}, \tag{2.1}$$

where

$$\mathcal{A}_j = \begin{pmatrix} 0 & \cdots & 0 & A_{1,j} \\ A_{2,j} & & & 0 \\ & \ddots & & \vdots \\ 0 & & A_{\omega,j} & 0 \end{pmatrix}, \qquad \mathcal{B}_j = \begin{pmatrix} 0 & B_{2,j} & & 0 \\ \vdots & & \ddots & \\ 0 & & & B_{\omega,j} \\ B_{1,j} & 0 & \cdots & 0 \end{pmatrix},$$

$$\mathcal{E}_j = \begin{pmatrix} 0 & \cdots & 0 & E_{1,j} \\ E_{2,j} & & & 0 \\ & \ddots & & \vdots \\ 0 & & E_{\omega,j} & 0 \end{pmatrix},$$

$$\mathcal{F}_j = \begin{pmatrix} 0 & F_{2,j} & & 0 \\ \vdots & & \ddots & \\ 0 & & & F_{\omega,j} \\ F_{1,j} & 0 & \cdots & 0 \end{pmatrix}, \qquad \mathcal{C}_j = \operatorname{diag}\Big(C_{1,j}, C_{2,j}, \ldots, C_{\omega,j}\Big),$$