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Variants of the Empirical Interpolation Method: Symmetric formulation, choice of norms and rectangular extension

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A B S T R A C T

The Empirical Interpolation Method (EIM) is a greedy procedure that constructs approximate representations of two-variable functions in separated form. In its classical presentation, the two variables play a non-symmetric role. In this work, we give an equivalent definition of the EIM approximation, in which the two variables play symmetric roles. Then, we give a proof for the existence of this approximation, and extend it up to the convergence of the EIM, and for any norm chosen to compute the error in the greedy step. Finally, we introduce a way to compute a separated representation in the case where the number of selected values is different for each variable. In the case of a physical field measured by sensors, this is useful to discard a broken sensor while keeping the information provided by the associated selected field.

1. Introduction

Consider a function $f : \mathcal{X} \times \mathcal{Y} \to \mathbb{R}$. The Empirical Interpolation Method (EIM) [1,2] is an offline/online procedure that provides an approximate representation $I_d(f)$ of f in separated form, where the integer d denotes the number of terms in the representation. The offline stage of the EIM consists in selecting some points $(x_1, \dots, x_d) \in \mathcal{X}^d$ and $(y_1, \dots, y_d) \in \mathcal{Y}^d$ in a greedy fashion, such that

$$x_{k+1} = \arg\max_{x \in \mathcal{X}} \left\| \left(f - I_k(f)\right)(x, \cdot) \right\|_{L^\infty(\mathcal{Y})}, \tag{1a}$$

$$y_{k+1} = \arg\max_{y \in \mathcal{Y}} \left| \left(f - I_k(f)\right)(x_{k+1}, y) \right|, \tag{1b}$$

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where $I_k(f)$ denotes the separated representation constructed with the k first points (x_l, y_l), $l \in \{1, \ldots, k\}$. The method is efficient when the error $f - I_d(f)$ is small for reasonably small values of d. Some functions $q_l(y)$, $l \in \{1, \ldots, d\}$ and a matrix B of size $d \times d$ depending on these points are also constructed, see [1] and [2] for details. The separated representation is then obtained as

$$I_d(f)(x, y) = \sum_{1 \le l \le d} \lambda_l(x) q_l(y), \tag{2}$$

where the functions $\lambda_l(x)$, $l \in \{1, \ldots, d\}$, solve the linear system $\sum_{m=1}^{d} B_{l,m} \lambda_m(x) = f(x, y_l)$, $l \in \{1, \ldots, d\}$. The function $I_d(f)$ satisfies the following interpolation property [2, Lemma 1]: for all $m \in \{1, \ldots, d\}$,

$$I_d(f)(x, y_m) = f(x, y_m), \quad \text{for all } x \in \mathcal{X}, \tag{3a}$$

$$I_d(f)(x_m, y) = f(x_m, y), \quad \text{for all } y \in \mathcal{Y}. \tag{3b}$$

In practice, the size d is not chosen a priori, and the greedy procedure stops when $|\left(f - I_k(f)\right)(x_{k+1}, y_{k+1})|$ is small enough. Define $\mathcal{U} := \{f(x, \cdot), x \in \mathcal{X}\}$. Elements of $\mathcal{U}$ are functions from $\mathcal{Y}$ to $\mathbb{R}$.

Theorem 1.1 (*Existence of the Decomposition, [2, Theorem 1]*)**.** *Assume that the interpolation points are chosen according to (1a)–(1b) and that $d < \dim \operatorname{span}(\mathcal{U})$. Then, the separated representation (2) is well-defined.*

In [3], it is observed that $I_d(f)$ can be rewritten in an algebraically equivalent form as

$$I_d(f)(x, y) = \sum_{1 \le l,m \le d} D_{l,m} f(x_l, y) f(x, y_m), \tag{4}$$

where the matrix D depends on the points x_l, y_m, $l, m \in \{1, \ldots, d\}$, and can be constructed recursively during the offline stage of the EIM. It is easy to check that (3a)–(3b) is satisfied if and only if $D = F^{-T}$, where $F_{l,m} = f(x_l, y_m)$, which motivates an alternative presentation of the EIM based on Eq. (4) where the variables x and y play symmetric roles. The double summation in (4) emphasizes this symmetric role; note that, for instance, $I_d(f)(x, y) = \sum_{1 \le l \le d} \tilde{\lambda}_l(x) \tilde{q}_l(y)$, with $\tilde{\lambda}_l(x) = \sum_{1 \le m \le d} D_{l,m} f(x, y_m)$ and $\tilde{q}_l(y) = f(x_l, y)$.

Let $\|\cdot\|_{\mathcal{Y}}$ be a norm on $\mathcal{Y}$ and suppose that the interpolation points are now selected as

$$x_{k+1} = \arg\max_{x \in \mathcal{X}} \|\left(f - I_k(f)\right)(x, \cdot)\|_{\mathcal{Y}}, \tag{5a}$$

$$y_{k+1} = \arg\max_{y \in \mathcal{Y}} |\left(f - I_k(f)\right)(x_{k+1}, y)|, \tag{5b}$$

the difference with (1a)–(1b) being the arbitrary choice for the norm $\|\cdot\|_{\mathcal{Y}}$ in the first line, instead of just $\|\cdot\|_{L^\infty(\mathcal{Y})}$. One can exchange the roles of x and y in the previous algorithm, leading to

$$y_{k+1} = \arg\max_{y \in \mathcal{Y}} \|\left(f - I_k(f)\right)(\cdot, y)\|_{\mathcal{X}}, \tag{6a}$$

$$x_{k+1} = \arg\max_{x \in \mathcal{X}} |\left(f - I_k(f)\right)(x, y_{k+1})|, \tag{6b}$$

for an arbitrary norm $\|\cdot\|_{\mathcal{X}}$ on $\mathcal{X}$. In general, the couple (x_{k+1}, y_{k+1}) resulting from (6a)–(6b) differs from the one obtained with (5a)–(5b). Choosing the L^∞-norm in $\mathcal{Y}$ and $\mathcal{X}$ in (5a) and (6a) respectively, we infer that

$$(x_{k+1}, y_{k+1}) = \arg\max_{(x,y) \in \mathcal{X} \times \mathcal{Y}} |\left(f - I_k(f)\right)(x, y)|,$$

thus recovering the choice made in (1a)–(1b). For this specific choice of L^∞-norms on $\mathcal{X}$ and $\mathcal{Y}$, (5a)–(5b) is actually equivalent to (6a)–(6b).

The first contribution of this work is the following result: