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Symmetry analysis and rogue wave solutions for the $(2+1)$-dimensional nonlinear Schrödinger equation with variable coefficients

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ABSTRACT

This paper addresses $(2+1)$-dimensional nonlinear Schrödinger equation (NLSE). For the special case, linear Schrödinger equation (LSE), it can be transformed into the same form of equation. On the basis of different gauge constraint, we construct potential symmetries for the LSE. And then, we consider $(2+1)$-dimensional NLSE using Lie symmetry analysis. By means of similarity transformations, we study the $(2+1)$-dimensional NLSE with nonlinearities and potentials depending on time as well as on the spatial coordinates. At last, we present the rouge wave solutions of $(2+1)$-dimensional NLSE.

1. Introduction

The famous nonlinear Schrödinger equation (NLSE) comes up in many science fields. There are various versions of NLSE that are used to explain the complex physical phenomenon. As one of the most key models of nonlinear mathematical physics, they have attracted much attention due to their potential applications. In addition, soliton solutions play a key role in study of NLSE. It is well-known that nonlinear and dispersive effects generate soliton. They are investigated by many authors in different fields, such as in mean-field theory of Bose–Einstein condensates, nonlinear optics and other fields [1–5]. There are a lot of papers that handled the $(1+1)$-dimensional nonlinear Schrödinger equation. In Ref. [6], the authors deal with nonlinear Schrödinger equation with spatially inhomogeneous nonlinearities using Lie group method and canonical transformations. In Ref. [7], the authors again, using Lie group method and canonical transformations, studied quintic nonlinear Schrödinger equations with spatially inhomogeneous nonlinearities. In Ref. [8], the authors, based on the similarity transformations, investigated quintic nonlinear Schrödinger equation with time and space modulated nonlinearities and potentials. The authors in [9] obtained the new classes of

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(1 + 1)-dimensional nonlinear Schrödinger equation with time-dependent potentials, and then, they got the free particle equation via non-local transformations.

Compared with the situation of (1 + 1)-dimensional NLSE, the (2 + 1)-dimensional NLSE has been paid minor treatment. Whether from a mathematics or from a physics point of view, the (2 + 1)-dimensional NLSE or more high dimensional NLSE may provide more backgrounds for real world. Also, there are many methods to deal with the nonlinear evolution equations (NLEEs), such as group method [9–19], generalized multi-symplectic method [20–27] and so on. It is known that the notable symmetry of the infinite dimensional Hamiltonian system, named as the multi-symplectic structure [20–23] was generalized to the non conservative Hamiltonian dynamic system, which is named as generalized multi-symplectic method [24], and was applied in some applied mechanics problems, such as the oscillating problems of the carbon nanotube [25,26] and the competition between dispersion and dissipation in the dynamic system [27]. In addition, there are a lot of papers and excellent books [14,16] that are used to handle NLEEs using group method. The group method provided a systematic route to deal with NLEEs [14,16].

In this paper, we study the following (2 + 1)-dimensional NLSE using group method

$$iu_t + u_{xx} + u_{yy} - g(x, y, t)|u|^2u - V(x, y, t)u = 0, \tag{1}$$

where the first term gives the evolution term, the group velocity dispersion are given by the second and third term, $g(x, y, t)$ represents the coefficient of nonlinear term, and $V(x, y, t)$ is an external potential. Recently, the authors [10] studied the equivalence group of LSE. The authors in paper [11] presented the admissible transformations and normalized classes of NLSE. In Ref. [28], the authors considered the AB and KA soliton solutions, they also consider the mechanism for controlling the obtained localized solutions. They got the bright and dark soliton solutions in 2D graded-index waveguides [29].

The paper is divided in the following manner. In Section 2, we consider the special case, and construct the potential symmetries. In Section 3, we employ Lie group to study (1). In Section 4, we use the similarity transformations to construct explicit rouge wave solutions of the equation. We also give some examples. Conclusions are presented in the last section.

2. Special case: nonlinearity $g(x, y, t) = 0$.

In this section, we consider the special case, that is to say,

$$iu_t + u_{xx} + u_{yy} - V(x, y, t)u = 0. \tag{2}$$

Consider the following point transformation

$$\tau = T(t), \qquad X = \xi(x, t, u), \qquad Y = \eta(y, t, u), \quad u = U(x, y, t, u). \tag{3}$$

They map (2) into the same form of equation as follows

$$iU_T + U_{\xi\xi} + U_{\eta\eta} - V_1(\xi, \eta, T)U = 0 \tag{4}$$

and we get

$$\begin{aligned} &\xi = a_1(t)x + b_1(t), \qquad \eta = a_1(t)y + c_1(t), \qquad T = \int^t a_1(\mu)^2 d\mu, \\ &U = \mathrm{e}^{i\left[\frac{a_{1t}}{4a}(x^2+y^2)+\frac{b_{1t}}{2a}x+\frac{c_{1t}}{2a}y+d_1(t)\right]}u, \end{aligned} \tag{5}$$

and

$$\begin{aligned} V_1(\xi, \eta, T) = \frac{1}{a_1^2}\Bigg[&V(x, y, t) + \frac{2a_{1t}^2 - a_{tt}a_1}{4a_1^2}(x^2 + y^2) + \frac{2a_{1t}b_{1t} - a1tb_{1tt}}{2a_1^2}x \\ &+ \frac{2a_{1t}c_{1t} - a1tc_{1tt}}{2a_1^2}y + \frac{b_{1t}^2}{4a_1^2} + \frac{c_{1t}^2}{4a_1^2} + i\frac{a_{1t}}{a_1} - d_{1t}\Bigg], \end{aligned} \tag{6}$$