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Transmission problem of Schrödinger and wave equation with viscous damping

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ABSTRACT

In this paper, we consider the transmission problem of a Schrödinger equation with a viscous damped wave equation which acts as a controller of the system. We show that the system operator generates a C_0-semigroup of contractions in the energy state space, and the system is well-posed. By giving the asymptotic expressions of the eigenvalues of the system, we know they all locate in the left hand side of the complex plane. It follows that the C_0-semigroup generated by the system operator achieves strong stability when the feedback gain is real.

1. Introduction

In the past few decades, there have been extensive literature on the control of Schrödinger equation (see [1–4]). In [2], the collocated boundary control is designed to exponentially stabilize the Schrödinger system

$$\begin{cases} w_t(x,t) + iw_{xx}(x,t) = 0, & x \in (0,1),\ t > 0, \\ w_x(1,t) = 0, & t \geq 0, \\ w(0,t) = U(t), & t \geq 0, \\ Y(t) = w_x(0,t), & t \geq 0, \end{cases} \tag{1.1}$$

where $U(t)$ is the control input and $Y(t)$ is the output observation. When $U(t) = -icY(t)$, where $c > 0$ is a positive constant, the authors in [2] showed that the system operator of the closed-loop system generates an exponentially stable semigroup in the energy space; and the eigenvalues approach a vertical line parallel to the imaginary axis. It is also known that, there has been much interest in studying the transmission problems, that is, the vibrational propagation over bodies consisting of two physically different types of

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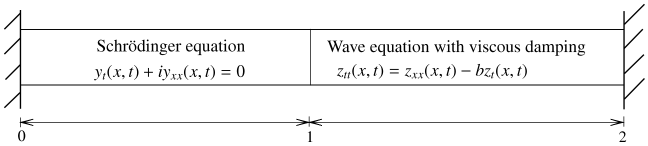

Fig. 1. Block diagram for the transmission between Schrödinger–Wave system.

materials [5,6]. The transmission problem to hyperbolic equations was studied in [5], the authors proved the existence and regularity of solutions for the linear problem. In [6], the authors study the wave propagations over materials consisting of elastic and viscoelastic components, and they proved that the dissipation produced by the viscoelastic part is strong enough to ensure the decay of the solution.

We are thus inspired and interested in studying the transmission problem of the Schrödinger equation with a viscous damped wave equation, to show through vibration and energy exchanging in the intersection, the whole system could finally be stable. In other words, we replace the static feedback in [2] by dynamic feedback governed by a wave equation with viscous damping and view it as a controller. The transmission problem of Schrödinger–wave system (as shown in Fig. 1) is written as follows:

$$\begin{cases} y_t(x,t) + iy_{xx}(x,t) = 0, & 0 < x < 1,\ t > 0, \\ z_{tt}(x,t) = z_{xx}(x,t) - bz_t(x,t), & 1 < x < 2,\ t > 0, \\ y(0,t) = z(2,t) = 0, & t \geq 0, \\ y(1,t) = kz_t(1,t), & t \geq 0, \\ z_x(1,t) = iky_x(1,t), & t \geq 0, \end{cases} \tag{1.2}$$

where $b > 0$. The two equations are connected at $x = 1$ and fixed at each end.

By introducing the following transformation

$$\begin{cases} w(x,t) = y(1-x,t), & 0 < x < 1,\ t > 0, \\ u(x,t) = z(x+1,t), & 0 < x < 1,\ t > 0, \end{cases} \tag{1.3}$$

then (1.2) becomes

$$\begin{cases} w_t(x,t) + iw_{xx}(x,t) = 0, & 0 < x < 1,\ t > 0, \\ u_{tt}(x,t) = u_{xx}(x,t) - bu_t(x,t), & 0 < x < 1,\ t > 0, \\ w(1,t) = u(1,t) = 0, & t \geq 0, \\ w(0,t) = ku_t(0,t), & t \geq 0, \\ u_x(0,t) = ikw_x(0,t), & t \geq 0. \end{cases} \tag{1.4}$$

The energy function for (1.4) is given by

$$E(t) = \frac{1}{2}\int_0^1 \left[|w(x,t)|^2 + |u_x(x,t)|^2 + |u_t(x,t)|^2\right] dx. \tag{1.5}$$

In this paper, we analyze the spectrum for the connected system (1.4). The system is well-posed and we give the asymptotic expressions of the eigenvalues. We can see that this kind of design is effective, since it moves the eigenvalues of the Schrödinger and wave equation into the left hand side of the complex plane. Finally, the strong stability of the system is achieved.

The rest of this paper is organized as follows. Section 2 is devoted to present the well-posedness of the system. In Section 3 we give the spectral and stability analysis of the system. The conclusion remarks are shown in Section 4.