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Square-mean almost periodic solutions to some singular stochastic differential equations

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ABSTRACT

This paper is aimed at studying the existence of square-mean almost periodic solutions to some singular stochastic differential equations with square-mean almost periodic coefficients. Using the block diagonal form of a linear operator on the suitable space, we establish an existence result under some appropriate assumptions.

1. Introduction

Let $M(m,\mathbb{C})$ be the vector space of all $m \times m$ − square matrices with complex entries and let $(\mathbb{H}, \|\cdot\|)$ be a separable Hilbert space.

In a recent paper by Diagana et al. [1], the existence and uniqueness of almost periodic solutions to the deterministic singular systems of differential equations given by

$$Adx(t) + Bx(t)\,dt = h(t)\,dt \quad \text{for all } t \in \mathbb{R}, \tag{1.1}$$

where $A, B \in M(m,\mathbb{C})$ (possibly singular) and $h : \mathbb{R} \longrightarrow \mathbb{C}^m$ ($\mathbb{C}^m$ being the m-dimensional complex space), was investigated.

Considering the fact that noise or stochastic perturbation is unavoidable and omnipresent in nature as well as in most of man-made phenomena; in this paper we initiate the study of the following singular stochastic

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differential equation with square-mean almost periodic coefficients

$$Adx(t) + Bx(t)\,dt = f(t)\,dt + g(t)\,dW(t) \quad \text{for all } t \in \mathbb{R}, \tag{1.2}$$

where $A : D(A) \subset \mathbb{H} \mapsto \mathbb{H}$ and $B : D(B) \subset \mathbb{H} \mapsto \mathbb{H}$ are closed linear operators such that $D(B) \subseteq D(A)$, $f, g : \mathbb{R} \longrightarrow L^2(\Omega, \mathbb{H})$ are square-mean almost periodic stochastic processes satisfying some additional conditions, $W(t)$ stands for a two-sided and standard one-dimensional Brownian motion defined on the filtered probability space $(\Omega, \mathcal{F}, P, \mathcal{F}_t)$ with $\mathcal{F}_t = \sigma\{W(u) - W(v) | u, v \le t\}$ with $L^2(\Omega, \mathbb{H})$ being the space of all $\mathbb{H}$-valued random variables x such that

$$\mathbb{E}\,\|x\|^2 = \int_\Omega \|x\|^2\,dP < \infty.$$

In order to study the existence of square-mean almost periodic solutions to Eq. (1.2), we make extensive use of techniques developed by Favini et al. [2]. The existence of almost periodic solutions to stochastic differential equations is an important topic of study essentially due to applications in physics, statistics, mechanics and mathematical biology. Various investigations on this topic have recently been made, see, e.g., [3–6]. In this paper it goes back to studying the existence of square-mean mild solutions to Eq. (1.2).

The paper is organized as follows. Section 2 is devoted to almost periodic processes. In Section 3, using techniques developed by Favini et al. [2], we formulate suitable assumptions and establish the existence of square-almost periodic solutions to Eq. (1.2).

2. Square-mean almost periodic processes

Definition 2.1 *([7])*. Let $x : \mathbb{R} \longrightarrow L^2(\Omega, \mathbb{H})$ be a stochastic process.

(1) x is said to be stochastically bounded if there exists $M > 0$ such that

$$\mathbb{E}\|x(t)\|^2 \le M \quad \text{for all } t \in \mathbb{R}.$$

(2) x is said to be stochastically continuous if

$$\lim_{t \to s} \mathbb{E}\|x(t) - x(s)\|^2 = 0 \quad \text{for all } s \in \mathbb{R}.$$

In this paper, $BC(\mathbb{R}, L^2(\Omega, \mathbb{H}))$ stands for the space of all the stochastically bounded and continuous processes. Clearly, the space $BC(\mathbb{R}, L^2(\Omega, \mathbb{H}))$ is a Banach space when it is equipped with the norm:

$$\|x\|_\infty := \sup_{t \in \mathbb{R}} \left(\mathbb{E}\|x(t)\|^2 \right)^{\frac{1}{2}}.$$

Definition 2.2 *([7])*. A continuous stochastic process $x : \mathbb{R} \longrightarrow L^2(\Omega, \mathbb{H})$ is said be square-mean almost periodic process if for each $\varepsilon > 0$ there exists $l(\varepsilon) > 0$ such that for all $\alpha \in \mathbb{R}$, there exists $\tau \in [\alpha, \alpha + l]$ satisfying $\sup_{t \in \mathbb{R}} \mathbb{E}\|x(t + \tau) - x(t)\|^2 < \varepsilon$.

The collection of all square-mean almost periodic processes defined above will be denoted by $AP(\mathbb{R}, L^2(\Omega, \mathbb{H}))$. It is well-known that $AP(\mathbb{R}, L^2(\Omega, \mathbb{H}))$ equipped with the sup-norm $\|\cdot\|_\infty$ is a Banach space.