



New oscillation criterion for delay differential equations with non-monotone arguments



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ABSTRACT

We investigate the oscillation of a first order delay differential equation with non-negative coefficient and non-monotone arguments. New oscillation criterion of *lim sup* type is obtained. An example is given to show the applicability and strength of the obtained condition over known ones.

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1. Introduction

This paper is devoted to study the oscillation of the first order delay differential equation

$$x'(t) + p(t)x(h(t)) = 0, \quad t \geq t_0, \quad (1)$$

where $p(t) \in C([t_0, \infty), [0, \infty))$ and $h(t)$ is a continuous function on $[t_0, \infty)$ with $h(t) \leq t$, such that $\lim_{t \rightarrow \infty} h(t) = \infty$.

By a solution of Eq. (1), we mean a continuous function $x(t)$ on $[t^*, \infty)$, where $t^* = \min_{t \geq t_0} h(t)$, which is continuously differentiable on $[t_0, \infty)$ and satisfies Eq. (1) for all $t \geq t_0$. A solution is called oscillatory if it has arbitrarily large zeros; otherwise it is called nonoscillatory. Equation (1) is called oscillatory if all its solutions are oscillatory; otherwise it is called nonoscillatory.

Since the pioneering work of Myshkis [1], many interesting oscillation criteria have been obtained for Eq. (1), see for example the papers [1–19] and the references cited therein as well as the monographs [20,21]. Each of the following celebrated conditions

$$k = \liminf_{t \rightarrow \infty} \int_{h(t)}^t p(s) ds > \frac{1}{e}, \quad (2)$$

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and

$$L = \limsup_{t \rightarrow \infty} \int_{h(t)}^t p(s) ds > 1, \quad (3)$$

due to [8] and [12] respectively; is sufficient for the oscillation of Eq. (1) such that $h(t)$ is nondecreasing when (3) holds. It is also well known, see [8], that Eq. (1) is nonoscillatory when $L < \frac{1}{e}$. Moreover, a simple consequence of Braverman and Karpuz [2, Theorem 1], shows that (3) is not sufficient for the oscillation of Eq. (1) if h is not monotonic. Therefore oscillation (or nonoscillation) criteria that cover the case when $k \leq \frac{1}{e} < L < 1$ are highly important, in particular if h is not monotonic. An account of such results can be found in [2,7,9,16,17].

In this work, we obtain new oscillation criterion of *lim sup* type for Eq. (1) which not only generalizes (3) but also works well when $k \leq \frac{1}{e} < L < 1$ regardless of the monotonicity pattern of the delay. So, our result can be applied to a wide range of equations of the form (1). An example is given to show that our result can test the oscillation of some equations while some known results fail to do so.

2. Main results

Throughout this work, we assume the existence of a nondecreasing continuous function $g(t)$ such that $h(t) \leq g(t) \leq t$, $t \geq t_1$ for some $t_1 \geq t_0$. Moreover, we define the sequences $\{S_n\}_{n \geq 0}$ and $\{q_n\}_{n \geq 0}$ respectively as follows:

$$\begin{aligned} S(t) &= \inf\{s \geq t_0 : h(s) > t\}, \quad \text{for } t \geq t_0, \\ S_1 &= S, \quad S_n = S \circ S_{n-1} \quad (n = 2, 3, \dots), \end{aligned}$$

where $\circ$ denotes the usual composition operation, and

$$\begin{aligned} q_0(t) &= p(t), \quad t \geq t_0, \\ q_1(t) &= q_0(t) \int_{h(t)}^t q_0(s) e^{\int_{h(s)}^t q_0(u) du} ds, \quad t \geq S_2(t_0), \\ q_n(t) &= q_{n-1}(t) \int_{g(t)}^t q_{n-1}(s) e^{\int_{g(s)}^t q_{n-1}(u) du} ds, \quad t \geq S_{2n}(t_1), \quad n = 2, 3, \dots \end{aligned}$$

Theorem 2.1. Let $\rho = \liminf_{t \rightarrow \infty} \int_{g(t)}^t p(s) ds$ and

$$D(\rho) := \begin{cases} 0, & \rho > \frac{1}{e}, \\ \frac{1 - \rho - \sqrt{1 - 2\rho - \rho^2}}{2}, & 0 \leq \rho \leq \frac{1}{e}. \end{cases}$$

If there exists a non-negative integer n such that

$$\limsup_{t \rightarrow \infty} \left(\int_{g(t)}^t q_n(s) ds + D(\rho) e^{\int_{g(t)}^t \sum_{i=0}^{n-1} q_i(s) ds} \right) > 1, \quad (4)$$

where $\sum_{i=0}^{n-1} q_i = 0$, then Eq. (1) is oscillatory.

Proof. Let $x(t)$ be a nonoscillatory solution of Eq. (1). Without loss of generality, we assume that $x(t)$ is eventually positive. Then there exists a sufficiently large $T > t_1$ such that $x(t) > 0$ for all $t > T$. This implies that $x(t)$ is nondecreasing for all $t > S_1(T)$, so $x(h(t)) \geq x(g(t))$, for all $t > S_2(T)$. Therefore, Eq. (1) leads to

$$x'(t) + q_0(t)x(g(t)) \leq 0, \quad t > S_2(T). \quad (5)$$

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