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On the forced oscillation of certain fractional partial differential equations☆

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ABSTRACT

In this paper, by using the technique of the differential inequality, sufficient conditions are obtained for the forced oscillation of certain fractional partial differential equations. The main results are illustrated by some examples.

1. Introduction

It is well known that fractional differential equations are generalizations of classical differential equations to an arbitrary (non-integer) order. The many important mathematical models are described by differential equations containing fractional order derivatives. In recent years, the theory of fractional differential equations and their applications have been investigated extensively. We refer the reader to the literatures [1–4]. At the same time, some results on the oscillatory behavior of solutions of fractional ordinary differential equations were established. For example, see [5–11] and the references therein. However, to the best of author's knowledge very little is known regarding the oscillatory behavior of fractional partial differential equations up to now, we refer to [12–14].

Our aim in this paper is to study the forced oscillation of fractional partial differential equations of the form

$$D^{\alpha}_{+,t}u(x,t) = a(t)\Delta u(x,t) - m(x,t,u(x,t)) + f(x,t), \quad (x,t) \in \Omega \times R_+ \equiv G, \tag{1}$$

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where Ω is a bounded domain in R^n with a piecewise smooth boundary $\partial\Omega$, $R_+ = [0, \infty)$, $\alpha \in (0, 1)$ is a constant, $D^{\alpha}_{+,t}u(x, t)$ is the Riemann–Liouville fractional derivative of order α of u with respect to t, and $\Delta u(x, t) = \sum_{r=1}^{n} \frac{\partial^2 u(x,t)}{\partial x_r^2}$.

We assume throughout this paper that

(A1) $a \in C(R_+; (0, \infty))$;

(A2) $m \in C(\overline{G} \times R; R)$, and

$$m(x, t, \xi) \begin{cases} \geq 0, & \text{if } \xi \in (0, \infty), \\ \leq 0, & \text{if } \xi \in (-\infty, 0); \end{cases}$$

(A3) $f \in C(\overline{G}; R)$.

Consider the following boundary condition:

$$\frac{\partial u(x, t)}{\partial N} = \psi(x, t), (x, t) \in \partial\Omega \times R_+, \tag{2}$$

where N is the unit exterior normal vector to $\partial\Omega$ and $\psi(x, t)$ is a continuous function on $\partial\Omega \times R_+$.

By a solution of the problem (1), (2), we mean a function $u(x, t)$ which satisfies (1) on $\overline{G}$ and the boundary condition (2).

A solution $u(x, t)$ of the problem (1), (2) is said to be oscillatory in G if it is neither eventually positive nor eventually negative, otherwise it is nonoscillatory.

Definition 1.1. The Riemann–Liouville fractional partial derivative of order $0 < \alpha < 1$ with respect to t of a function $u(x, t)$ is given by

$$D^{\alpha}_{+,t}u(x, t) := \frac{\partial}{\partial t} \frac{1}{\Gamma(1 - \alpha)} \int_0^t (t - \nu)^{-\alpha} u(x, \nu) d\nu \tag{3}$$

provided the right hand side is pointwise defined on R_+, where Γ is the gamma function.

Definition 1.2. The Riemann–Liouville fractional integral of order $\alpha > 0$ of a function $y : R_+ \to R$ on the half-axis R_+ is given by

$$I^{\alpha}_{+}y(t) := \frac{1}{\Gamma(\alpha)} \int_0^t (t - \nu)^{\alpha-1} y(\nu) d\nu \quad \text{for } t > 0 \tag{4}$$

provided the right hand side is pointwise defined on R_+.

Definition 1.3. The Riemann–Liouville fractional derivative of order $\alpha > 0$ of a function $y : R_+ \to R$ on the half-axis R_+ is given by

$$\begin{aligned} D^{\alpha}_{+}y(t) &:= \frac{d^{\lceil\alpha\rceil}}{dt^{\lceil\alpha\rceil}} \left(I^{\lceil\alpha\rceil-\alpha}_{+} y \right)(t) \\ &= \frac{1}{\Gamma(\lceil\alpha\rceil - \alpha)} \frac{d^{\lceil\alpha\rceil}}{dt^{\lceil\alpha\rceil}} \int_0^t (t - \nu)^{\lceil\alpha\rceil-\alpha-1} y(\nu) d\nu \quad \text{for } t > 0 \end{aligned} \tag{5}$$

provided the right hand side is pointwise defined on R_+, where $\lceil\alpha\rceil$ is the ceiling function of α.

Lemma 1.1 (*[13]*). *Let*

$$E(t) =: \int_0^t (t - v)^{-\alpha} y(v) dv \quad \textit{for } \alpha \in (0, 1) \textit{ and } t > 0. \tag{6}$$

Then $E'(t) = \Gamma(1 - \alpha) D^{\alpha}_{+}y(t)$.