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Integrability of a (2+1)-dimensional generalized breaking soliton equation

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ABSTRACT

Under investigation in this letter is a (2+1)-dimensional generalized breaking soliton equation, which describes the (2+1)-dimensional interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis. A singularity analysis is carried out and it is shown that this generalized equation admits the Painlevé property for one set of parametric choices. Some integrable properties of the corresponding Painlevé integrable equation, such as its bilinear form, N-soliton solution, bilinear Bäcklund transformation, Lax pair and infinite conservation laws are derived with the binary Bell polynomials.

1. Introduction

The investigation of integrable properties for nonlinear evolution equations (NLEEs) plays an important role in the study of nonlinear phenomena. Painlevé analysis is a widely applied and quite successful technique to test the integrability of NLEEs by analyzing the singularity structure of the solutions [1,2]. The bilinear method developed by Hirota provides a powerful way to study the integrability and exact solutions of nonlinear equations [3]. Once a given nonlinear equation is written in bilinear form, one can obtain multi-soliton solutions, bilinear BT, Lax pair and so on. However, there is no universal method to get the bilinear forms and bilinear BT. Based on the use of Bell polynomials [4], Lambert and his coworkers have proposed a direct procedure to obtain bilinear forms, bilinear BT and Lax pairs for soliton equation in quite a systematic way [5,6]. Fan extended this method to the variable-coefficient and supersymmetric soliton equations [7,8]. Ma systematically studied the connections between Bell polynomials and bilinear representation [9,10]. In recent years, Bell polynomials approach is widely used to derive integrable properties for a large number of nonlinear equations [11–16].

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In this letter, we will employ the singularity analysis and binary Bell polynomials' approach to study the integrability for a $(2+1)$-dimensional generalized breaking soliton equation,

$$\begin{aligned} &u_t + au_{xxx} + bu_{xxy} + cuu_x + duv_x + eu_xv = 0, \\ &u_y = v_x, \end{aligned} \tag{1}$$

where a, b, c, d, e are real parameters. Much research has been done for the particular cases of Eq. (1). To the author's knowledge, the integrability of Eq. (1) with general form has not been considered.

Eq. (1) includes a lot of important NLEEs as its special cases. For example, if $u = w_x$, $v = w_y$, $a = c = 0$, $b = 1$, $d = -4$, $e = -2$, then this equation transforms into the breaking soliton equation

$$w_{xt} + w_{xxxy} - 2w_{xx}w_y - 4w_xw_{xy} = 0, \tag{2}$$

which was proposed by Calogero and Degasperis [17]. In particular, if $u = w_x$, $v = w_y$, $a = c = 0$, $b = 1$, $d = e = 4$, Eq. (1) reduces to another similar breaking soliton equation,

$$w_{xt} + w_{xxxy} + 4w_{xx}w_y + 4w_xw_{xy} = 0, \tag{3}$$

which has been studied by Bogoyavlenskii and the overlapping solutions have been given [18]. These two equations describe the $(2+1)$-dimensional interaction of a Riemann wave propagating along the y-axis with a long wave along the x-axis, several types of exact solutions have been constructed [19–22].

Moreover, if $c = 6a$, $d = e = 4b$, Eq. (1) reduces to the Bogoyavlensky–Konoplechenko equation

$$\begin{aligned} &u_t + au_{xxx} + bu_{xxy} + 6auu_x + 4buv_x + 4bu_xv = 0, \\ &u_y = v_x. \end{aligned} \tag{4}$$

The dromion-like structures of Eq. (4) with $a = 0$ have been derived [23]. For Eq. (4), a series of exact solutions such as positon, negaton, complexiton solutions have been constructed [24], while the Wronskian solution and bilinear BT have been studied in the viewpoint of Bell polynomials [25].

In the case $c = 6a$, $d = e = 3b$, Eq. (1) reduces to

$$\begin{aligned} &u_t + au_{xxx} + bu_{xxy} + 6auu_x + 3buv_x + 3bu_xv = 0, \\ &u_y = v_x, \end{aligned} \tag{5}$$

which has been studied by Zhang et al. and the quasi-periodic solutions have been given [26]. Ma et al. [27] constructed the one-periodic and two periodic wave solutions to Eq. (5) with $a = 0$.

The plan of this letter is as below. In Section 2, we carry out the singularity analysis for Eq. (1) to derive its integrable conditions. In Section 3, for the new integrable subcase of Eq. (1), its bilinear form, bilinear BT, Lax pair and infinite conservation laws are derived by using the binary Bell polynomials method. Finally, some conclusions are given in Section 4.

2. Painlevé analysis of Eq. (1)

Painlevé analysis is one of the most powerful methods to find the underlying integrable models from a given generalized NLEEs [28,29]. Following the WTC-Kruskal approach [1], the first step of the Painlevé test is the leading order analysis. One may suppose

$$u(x, y, t) = u_0\phi^{\alpha_1}, \qquad v(x, y, t) = v_0\phi^{\alpha_2}.$$

Inserting it into Eq. (1) and equating the most dominant terms, one obtains

$$\alpha_1 = \alpha_2 = -2, \qquad u_0 = -\frac{12\phi_x^2(a\phi_x + b\phi_y)}{c\phi_x + (d+e)\phi_y}, \qquad v_0 = -\frac{12\phi_x\phi_y(a\phi_x + b\phi_y)}{c\phi_x + (d+e)\phi_y}. \tag{6}$$