Applied Mathematics Letters 50 (2015) 91–97

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Boundedness and asymptotic behavior of solutions to a chemotaxis–haptotaxis model in high dimensions

Yuhuan Li[a], Ke Lin[b,*], Chunlai Mu[b]

[a] Department of Mathematics, Sichuan Normal University, Chengdu 610066, PR China
[b] College of Mathematics and Statistics, Chongqing University, Chongqing 401331, PR China

ARTICLE INFO

Article history:
Received 15 April 2015
Received in revised form 12 June 2015
Accepted 12 June 2015
Available online 20 June 2015

Keywords:
Chemotaxis
Haptotaxis
Boundedness
Asymptotic stability

ABSTRACT

We consider the Neumann value problem for the chemotaxis system

$$\begin{cases} u_t = \nabla \cdot \left(\nabla u - u \left(\dfrac{\alpha}{1+v} \nabla v + \rho \nabla w \right) \right) + \lambda u(1-u), & x \in \Omega,\ t > 0, \\ v_t = \Delta v - v - \mu u v, & x \in \Omega,\ t > 0, \\ w_t = \gamma u(1-w), & x \in \Omega,\ t > 0, \end{cases}$$

in a bounded domain $\Omega \subset R^n (n \geq 1)$ with smooth boundary, where $\alpha, \rho, \lambda, \mu$ and γ are positive coefficients. It is shown that for any choice of reasonably regular initial data (u_0, v_0, w_0), there exists a constant λ^* depending on $\alpha, \rho, \mu, \gamma, n, v_0$ and w_0 such that for any $\lambda > \lambda^*$, the associated initial–boundary system possesses a global classical solution which is uniformly bounded. Moreover, building on this boundedness property, it is proved that as time tends to infinity, all the solution approaches the homogeneous steady state $(1, 0, 1)$ in an appropriate sense.

1. Introduction

In recent year, some mathematical models for tumor growth have been developed and basic mathematical aspects related to these systems have been explored by many authors [1–6]. In this paper we focus on a mathematical model which describes the directed movements of the endothelial cells in response to a concentration gradient of a chemical signal (tumor angiogenic factor, TAF) secreted by tumor cells and in response to a concentration gradient of the fibronectin produced by the endothelial cells. The more details can be seen in [3]. More precisely, we denote the endothelial cells density by $u(x,t)$, the TAF density by

* Corresponding author.
E-mail address: shuxuelk@126.com (K. Lin).

http://dx.doi.org/10.1016/j.aml.2015.06.010
0893-9659/

$v(x,t)$ and the fibronectin density by $w(x,t)$ and consider the following system

$$\begin{cases} u_t = \nabla \cdot \left(\nabla u - u \left(\frac{\alpha}{1+v} \nabla v + \rho \nabla w \right) \right) + \lambda u(1-u), & x \in \Omega,\ t>0, \\ v_t = \Delta v - v - \mu u v, & x \in \Omega,\ t>0, \\ w_t = \gamma u(1-w), & x \in \Omega,\ t>0, \\ \frac{\partial u}{\partial \nu} - u \left(\frac{\alpha}{1+v} \frac{\partial v}{\partial \nu} + \rho \frac{\partial w}{\partial \nu} \right) = \frac{\partial v}{\partial \nu} = 0, & x \in \partial\Omega,\ t>0, \\ u(x,0)=u_0(x), \quad v(x,0)=v_0(x), \quad w(x,0)=w_0(x), & x \in \Omega \end{cases} \tag{1.1}$$

in a bounded domain $\Omega \subset R^n (n \geq 1)$, where throughout this paper we may assume that

$$\begin{cases} (u_0, v_0, w_0) \in (C^{2,\beta}(\bar{\Omega}))^3 \text{ is nonnegative with } u_0 \not\equiv 0 \text{ for some } \beta \in (0,1), \text{ and that} \\ \frac{\partial u_0}{\partial \nu} - u_0 \left(\frac{\alpha}{1+v_0} \frac{\partial v_0}{\partial \nu} + \rho \frac{\partial w_0}{\partial \nu} \right) = \frac{\partial v_0}{\partial \nu} = 0. \end{cases} \tag{1.2}$$

In one and two space dimensions, the global existence was investigated in [7] for any initial data (u_0, v_0, w_0) and the long-time behavior of solutions to (1.1) was established if either $w_0 > 1$, or $\|w_0 - 1\|_{L^\infty(\Omega)} < \delta$ for some $\delta > 0$. Since the nonlinear chemotactic sensitivity or the logistic term in (1.1) is known to make a somewhat stabilizing influence on the classical chemotaxis system in the sense of boundedness (see [8–12]), the global solvability of the parabolic–elliptic-ODE chemotaxis–haptotaxis model with logistic growth has been analyzed by various researchers, see for instance [13–16]. Moreover, similar statements have been also derived for the fully parabolic version of the system whenever the proliferation rate λ is suitably large (see [17,18]).

The purpose of this work is to establish the global solvability of solutions to (1.1) for the high-dimensional case and our results rule out any chemotactic collapse in (1.1) under some suitable largeness on λ.

Theorem 1.1. *Suppose that $\Omega \subset R^n (n \geq 1)$ is a bounded domain and let $\alpha, \rho, \lambda, \mu$ and γ be positive. Then for any choice of (u_0, v_0, w_0) fulfilling (1.2), there exists $\lambda^* = \lambda^*(\alpha, \rho, \mu, \gamma, n, \|v_0\|_{L^\infty(\Omega)}, \|w_0\|_{L^\infty(\Omega)})$ such that whenever $\lambda > \lambda^*$, the problem (1.1) possesses a unique global classical solution (u, v, w) which is uniformly bounded.*

The asymptotic behavior of solutions in (1.1) reads as follows.

Theorem 1.2. *Suppose that the assumptions in Theorem 1.1 are satisfied. Moreover, if*

$$\rho \|w_0 - 1\|_{L^\infty(\Omega)} < 1 \quad \text{or} \quad w_0 > 1, \tag{1.3}$$

then

$$u(\cdot,t) \to 1 \quad \text{in } L^2(\Omega), \qquad v(\cdot,t) \to 0 \quad \text{in } L^\infty(\Omega) \quad \text{and} \quad w(\cdot,t) \to 1 \quad \text{in } L^\infty(\Omega) \tag{1.4}$$

as $t \to \infty$.

Remark 1.1. If $\rho = 1$, Theorem 1.2 also holds for any $w_0 > 0$ if λ is large enough. However, we have to leave an open question here whether this is optimal in the sense that boundedness can be asserted for the case of some small λ for dimensions $n = 3$ or higher.

2. Boundedness

Let us first consider the local existence of solutions and give a criterion for their boundedness.

Lemma 2.1 ([7]). *Suppose that Ω is a bounded domain in $R^n (n \geq 1)$. For any initial data (u_0, v_0, w_0) satisfying (1.2), there exist $T_{\max} \in (0, \infty]$ and a unique nonnegative function*