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Global exponential stability of a delay reduced SIR model for migrant workers' home residence

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The paper is concerned with a reduced SIR model for migrant workers. By using differential inequality technique and a novel argument, we derive a set of conditions to ensure that the endemic equilibrium of the model is globally exponentially stable. The obtained results complement with some existing ones. We also use numerical simulations to demonstrate the theoretical results.

1. Introduction

In 2012, to describe the influence of temporary migration on the transmission of an infectious disease in a migrant workers' home residence, Wang and Wang [1] proposed the following delay differential system:

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - (\mu_S + m_S)S(t) - \beta S(t)I(t) + (1-p)m_S e^{-\delta\tau} S(t-\tau), \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - (\mu_I + \gamma)I(t) + p m_S e^{-\delta\tau} S(t-\tau), \\ \dfrac{dR(t)}{dt} = \gamma I(t) - (\mu_R + m_R)R(t) + m_R e^{-\delta\tau} R(t-\tau), \end{cases} \tag{1}$$

where $t \geq t_0 \geq 0$. As a classical Susceptible–Infected–Recovered (SIR) model, they assumed that the total size $N(t)$ was divided into three disjoint classes denoted by $S(t), I(t)$ and $R(t)$, respectively at time t; all parameters Λ, μ_S, μ_I, μ_R, β, γ, δ, τ, m_S and m_R, are positive constants; $p = p(\tau)$ denotes a probability of

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a migrant worker returning home, and $p(\tau)$ is increasing on $[0,+\infty)$ with $p(0)=0$ and $\lim_{\tau\to+\infty} p(\tau)=1$. Detailed biological explanations on model (1) can be found in [1].

Because the third equation is independent of the first two equations in system (1), we only need to study the following reduced system:

$$\begin{cases} \dfrac{dS(t)}{dt} = \Lambda - (\mu_S + m_S)S(t) - \beta S(t)I(t) + (1-p)m_S e^{-\delta\tau} S(t-\tau), \\ \dfrac{dI(t)}{dt} = \beta S(t)I(t) - (\mu_I + \gamma)I(t) + p m_S e^{-\delta\tau} S(t-\tau). \end{cases} \tag{2}$$

Let $\mathbb{R}_+$ denote the nonnegative real number space, $C = C([-\tau,\ 0],\mathbb{R})$ be the continuous function space equipped with the usual supremum norm $\|\cdot\|$, and let $C_+ = C([-\tau,0],\mathbb{R}_+)$. If $x(t)$ is continuous on $[-\tau+t_0,\ \sigma)$ with $t_0,\sigma\in\mathbb{R}$ and $t_0<\sigma$, then we define $x_t\in C$ where $x_t(\theta)=x(t+\theta)$ for all $\theta\in[-\tau,0]$. In this paper, we just consider system (2) with the following initial conditions:

$$S_{t_0} = \varphi, \quad S(t_0) > 0,\ \varphi\in C_+,\ I(t_0)\in\mathbb{R}_+\backslash\{0\}. \tag{3}$$

By using the mathematical and analytic technique, Wang and Wang [1] and Sigdel and McCluskey [2] proposed several strategies to control the spread of infectious diseases. Also, the existence and global asymptotical stability of the unique positive equilibrium (S^*, I^*) of system (2) with the initial value (3) were proved in [1,2], where (S^*, I^*) satisfied

$$\begin{cases} 0 = \Lambda - (\mu_S + m_S)S^* - \beta S^* I^* + (1-p)m_S e^{-\delta\tau} S^*, \\ 0 = \beta S^* I^* - (\mu_I + \gamma)I^* + p m_S e^{-\delta\tau} S^*. \end{cases} \tag{4}$$

Furthermore, the results in [1,2] imply that the disease will always eventually reach a positive constant level in the population and

$$\lim_{t\to+\infty}(S(t), I(t)) = (S^*, I^*). \tag{5}$$

On the other hand, the convergence speed of $\lim_{t\to+\infty}(S(t),I(t)) = (S^*,I^*)$ is vitally important to disease prevention and control. In particular, since the exponential convergent rate can be unveiled, there have been extensive results on the problem of the exponential stability of population and ecology models in the literature. We refer the reader to [3–7] and the references cited therein. In fact, the exponential stability of (S^*, I^*) implies that the spread time and controlling time (the time required from the outbreak of the disease to close to a steady state) are much shorter than those of the asymptotical stability. Therefore, **a natural question is:** whether we can find conditions guaranteeing the exponential stability of the unique equilibrium (S^*, I^*) of system (2) with initial value conditions (3). This is the purpose of this paper.

2. Preliminaries and the main result

To get the main result, similar to Ref. [8], we give the definition of the exponential stability as follows.

Definition 1. Let $(S(t), I(t))$ be the solution of (2) with initial value conditions (3). If there exists a positive constant λ such that

$$|S(t) - S^*| = O(e^{-\lambda t}) \quad \text{and} \quad |I(t) - I^*| = O(e^{-\lambda t}) \quad \text{as } t\to+\infty,$$

then the above equilibrium (S^*, I^*) is said to be globally exponentially stable.

From (4), we easily obtain that $[\mu_S + m_S(1-e^{-\delta\tau})]S^* + (\mu_I+\gamma)I^* = \Lambda$, which means

$$S^* < \frac{\Lambda}{\mu_S + m_S(1-e^{-\delta\tau})} \quad \text{and} \quad I^* < \frac{\Lambda}{\mu_I+\gamma}.$$

From the above inequalities and (5), we have the following lemma: