



Existence of positive solutions for n th-order p -Laplacian singular super-linear boundary value problems



Zhongli Wei*

Department of Mathematics, Shandong Jianzhu University, Jinan, Shandong, 250101, People's Republic of China

School of Mathematics, Shandong University, Jinan, Shandong 250100, People's Republic of China

ARTICLE INFO

Article history:

Received 29 April 2015

Received in revised form 25 June 2015

Accepted 25 June 2015

Available online 7 July 2015

Keywords:

Singular super-linear boundary value problem

Positive solution

Fixed point theorem

Cone

n th-order p -Laplacian

ABSTRACT

This paper investigates the existence of positive solutions for n th-order ($n \geq 3$) p -Laplacian singular super-linear boundary value problems. A necessary and sufficient condition for the existence of S_p^{n-2} positive solutions as well as S_p^{n-1} positive solutions is given by means of the fixed point theorems on cones.

© 2015 Elsevier Ltd. All rights reserved.

1. Introduction

The singular ordinary differential equations arise in the fields of gas dynamics, Newtonian fluid mechanics, the theory of boundary layer and so on, the theory of singular boundary value problems has become an important area of investigation in recent years (see [1–4] and the references therein). Consider the n th-order ($n \geq 3$) p -Laplacian singular super-linear boundary value problems

$$\begin{cases} \varphi_p(x^{(n-2)})''(t) + f(t, x(t), x'(t), \dots, x^{(i)}(t), \dots, x^{(n-2)}(t)) = 0, & t \in (0, 1), \\ x^{(i)}(0) = 0, & i = 0, 1, 2, \dots, n-2, \quad x^{(n-2)}(1) = 0, \end{cases} \quad (1.1)$$

where $\varphi_p(t) = |t|^{p-2}t$, $p \geq 2$, $n \geq 3$ is an integer, and f satisfies the following hypothesis.

* Correspondence to: Department of Mathematics, Shandong Jianzhu University, Jinan, Shandong, 250101, People's Republic of China.

E-mail address: jnwzl32@163.com.

(H): $f \in C((0, 1) \times [0, \infty)^{n-1}, [0, \infty))$, and there exist constants λ_i, μ_i , ($0 < \lambda_i \leq \mu_i, i = 0, 1, 2, \dots, n-3, 0 \leq \lambda_{n-2} \leq \mu_{n-2} < p-1, \sum_{i=0}^{n-2} \lambda_i > p-1$) such that for $t \in (0, 1), x_i \in [0, \infty)$

$$\begin{aligned} c^{\mu_i} f(t, x_0, x_1, \dots, x_i, \dots, x_{n-2}) &\leq f(t, x_0, x_1, \dots, cx_i, \dots, x_{n-2}) \\ &\leq c^{\lambda_i} f(t, x_0, x_1, \dots, x_i, \dots, x_{n-2}), \quad \text{if } 0 < c \leq 1, \quad i = 0, 1, 2, \dots, n-2. \end{aligned} \quad (1.2)$$

Remark 1.1. (1.2) is equivalent to the following (1.3)

$$\begin{aligned} c^{\lambda_i} f(t, x_0, x_1, \dots, x_i, \dots, x_{n-2}) &\leq f(t, x_0, x_1, \dots, cx_i, \dots, x_{n-2}) \\ &\leq c^{\mu_i} f(t, x_0, x_1, \dots, x_i, \dots, x_{n-2}), \quad \text{if } c \geq 1, \quad i = 0, 1, \dots, n-2. \end{aligned} \quad (1.3)$$

Typical functions that satisfy the above super-linear hypothesis are those taking the form $f(t, x_0, x_1, \dots, x_{n-2}) = \sum_{i=1}^N p_i(t) x_0^{\ell_{i0}} x_1^{\ell_{i1}} \dots x_{n-2}^{\ell_{i,n-2}}$; here $p_j(t) \in C(0, 1), p_j(t) > 0$ on $(0, 1), 0 < \ell_{jk}, k = 0, 1, \dots, n-3, 0 \leq \ell_{j,n-2} < p-1, \sum_{k=0}^{n-2} \ell_{jk} > p-1, j = 1, 2, \dots, N$.

By singularity we mean that the function $f(t, x_0, x_1, \dots, x_i, \dots, x_{n-2})$ in (1.1) is allowed to be unbounded at $x_i, i = 0, 1, \dots, n-2, t = 0$ and or $t = 1$. Note $S_p^{n-2} = \{x \in C^{n-2}[0, 1], \varphi_p(x^{(n-2)}) \in C[0, 1] \cap C^2(0, 1)\}, S_p^{n-1} = \{x \in C^{n-1}[0, 1], \varphi_p(x^{(n-2)}) \in C^1[0, 1] \cap C^2(0, 1)\}$. A function $x(t) \in S_p^{n-2}$ is called a S_p^{n-2} (positive) solution of (1.1) if it satisfies (1.1) ($x^{(i)}(t) > 0, i = 0, 1, \dots, n-2$, for $t \in (0, 1)$). A function $x(t) \in S_p^{n-1}$ is called a S_p^{n-1} (positive) solution of (1.1) if $x^{(n-1)}(0^+)$ and $x^{(n-1)}(1^-)$ both exist, and it satisfies (1.1) ($x^{(i)}(t) > 0, i = 0, 1, \dots, n-2$, for $t \in (0, 1)$).

Using the Krasnosel'skii, Leggett–Williams fixed-point theorems, the authors in papers [5–11] have given some sufficient conditions for the existence of positive solution and multiple positive solutions for a class of n th-order m -point singular boundary value problems. By constructing lower and upper solutions and with the comparison theorem, Z.L. Wei in paper [12] has given a necessary and sufficient condition for the existence of S_p^{n-1} positive solutions of n th-order p -Laplacian singular sub-linear boundary value problems (1.1).

Now, in this paper, we shall give some necessary and sufficient conditions for the existence of S_p^{n-2} positive solutions as well as S_p^{n-1} positive solutions of the singular super-linear boundary value problems (1.1) by using the fixed point theorems on cones, which are different from that of [5–12].

2. Some lemmas

To obtain the main results, we need the following lemmas.

Lemma 2.1. Suppose that (H) holds. Let $x(t)$ be a S_p^{n-1} positive solution of (1.1). Then there are constants $I_1^{(i)}$ and $I_2^{(i)}$, $0 < I_1^{(i)} \leq I_2^{(i)}$, such that

$$I_1^{(n-2)}(t(1-t))^{1/(p-1)} \leq x^{(n-2)}(t) \leq I_2^{(n-2)}(t(1-t))^{1/(p-1)}, \quad t \in [0, 1], \quad (2.1)$$

$$I_1^{(i)} t^{n-2-i+1/(p-1)} \leq x^{(i)}(t) \leq I_2^{(i)} t^{n-2-i+1/(p-1)}, \quad t \in [0, 1], \quad i = 0, 1, 2, \dots, n-3. \quad (2.2)$$

Proof. For convenience, we note $(Fx)(t) = f(t, x(t), x'(t), \dots, x^{(i)}(t), \dots, x^{(n-2)}(t))$. Assume that $x(t)$ is a S_p^{n-1} positive solution of (1.1). Then both $\varphi_p[x^{(n-2)}]'(0)$ and $\varphi_p[x^{(n-2)}]'(1)$ exist. By integration of (1.1), we have $\int_0^1 (Fx)dt = \varphi_p[x^{(n-2)}]'(0) - \varphi_p[x^{(n-2)}]'(1) < \infty$. And $x(t)$ can be written as

$$x^{(n-2)}(t) = \left(\int_0^1 H(t, s)(Fx)(s)ds \right)^{1/(p-1)}, \quad t \in [0, 1], \quad (2.3)$$

$$x^{(i)}(t) = \int_0^t \frac{(t-s)^{(n-i-3)}}{(n-i-3)!} x^{(n-2)}(s)ds, \quad i = 0, 1, 2, \dots, n-3, \quad t \in [0, 1], \quad (2.4)$$

Download English Version:

<https://daneshyari.com/en/article/1707632>

Download Persian Version:

<https://daneshyari.com/article/1707632>

[Daneshyari.com](https://daneshyari.com)