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Blow-up for wave equation with weak boundary damping and source terms

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ABSTRACT

In this paper, we consider the wave equation with nonlinear boundary damping and source terms. This work is devoted to prove a finite time blow-up result under suitable condition on the initial data and positive initial energy. The main goal of the present paper is to generalize our previous result in Ha (2012) treating the boundary damping term in a more general setting.

1. Introduction

In this paper, we are concerned with the blow-up of solutions for the wave equation:

$$\begin{cases} u'' - \mu(t)\Delta u + h(u) = 0 & \text{in } \Omega \times (0, +\infty), \\ u = 0 & \text{on } \Gamma_0 \times (0, +\infty), \\ \mu(t)\dfrac{\partial u}{\partial \nu} + g(u') = |u|^\gamma u & \text{on } \Gamma_1 \times (0, +\infty), \\ u(x,0) = u_0(x), \qquad u'(x,0) = u_1(x), & \end{cases} \tag{1.1}$$

where Ω is a bounded domain of $\mathbb{R}^n (n \geq 1)$ with boundary $\Gamma = \Gamma_0 \cup \Gamma_1$ of class C^2. Here, $\Gamma_0 \neq \emptyset$, Γ_0 and Γ_1 are closed and disjoint. Let ν be the outward normal to Γ. Δ stands for the Laplacian with respect to the spatial variables respectively, $'$ denotes the derivative with respect to time t.

The problem of proving the nonexistence and blow-up of solutions of the wave equation has been studied by many authors. In particular, there are many results with negative initial energy (see [1–8] and a list of references therein) also these results were obtained with convexity method. However much less is known when the initial energy is positive (cf. [9–13]) and these results used several other method for example, contradiction method, decomposition method and so on.

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All of above mentioned references have Dirichlet's boundary condition. On the other hand, there were very few results for problems with nontrivial boundary conditions. Vitillaro [14] considered the wave equation with linear interior–boundary damping terms and nonlinear boundary source term. He proved global nonexistence for the problem using convexity method, but did not prove blow-up result. Cavalcanti et al. [15] studied blow-up for the linear wave equation with boundary damping and interior–boundary source terms under suitable condition. Recently, our previous result [16] proved blow-up of solutions for the semilinear wave equation with positive initial energy by using potential well theory and contradiction method. This paper considered nonlinear boundary damping term having polynomial growth.

The most important terms determining the blow-up are damping term and source term, and state of solutions, that is blow-up or existence depends on the relation of damping and source terms. So, when source term has polynomial growth, we may be able to analyze the solutions if damping term also has polynomial growth. However, to my knowledge, there is no blow-up result without polynomial growth near zero assumption on the damping term. The goal of this paper is to generalize the result of [16] under a weaker assumption, that the damping term g does not necessarily have polynomial growth near zero.

This paper is organized as follows : In Section 2, we recall the notation, hypotheses and some necessary preliminaries and introduce our main result. In Section 3, we prove the blow-up of solutions for (1.1) by employing contradiction method.

2. Preliminaries

We begin this section by introducing some hypotheses and our main result. Throughout this paper, we use standard functional spaces with $\|\cdot\|_p$, $\|\cdot\|_{p,\Gamma_1}$ denote the $L^p(\Omega)$ norm and $L^p(\Gamma_1)$ norm, respectively.

($\mathbf{H_1}$) Hypotheses on $\boldsymbol{\Omega}$.

Let $\Omega \subset \mathbb{R}^n$ be a bounded and connected domain, $n \geq 1$, with boundary $\Gamma = \Gamma_0 \cup \Gamma_1$ of class C^2. Here Γ_0 and Γ_1 are closed and disjoint, $\Gamma_0 \neq \emptyset$, satisfying the following condition:

$$\begin{aligned} &m(x) \cdot \nu(x) \geq \sigma > 0 \quad \text{on}\, \Gamma_1, \qquad m(x) \cdot \nu(x) \leq 0 \quad \text{on}\, \Gamma_0, \qquad m(x) = x - x^0 \quad (x^0 \in \mathbb{R}^n) \quad \text{and} \\ &R = \max_{x \in \overline{\Omega}} |m(x)|, \end{aligned} \tag{2.1}$$

where ν represents the unit outward normal vector to Γ. We assume that

$$\mu(0)\frac{\partial u_0}{\partial \nu} + g(u_1) = |u_0|^\gamma u_0 \quad \text{on}\, \Gamma_1. \tag{2.2}$$

($\mathbf{H_2}$) Hypotheses on $\boldsymbol{\mu}$, $\boldsymbol{h}$.

Let $\mu \in W^{2,\infty}(0,T) \cap W^{2,1}(0,T)$, where $0 < T < \infty$ can be taken arbitrarily large and

$$\mu(t) \geq \mu_0 > 0 \quad \text{and} \quad \mu'(t) < 0 \quad \text{a.e. in}\, (0,\infty). \tag{2.3}$$

Moreover, we assume that

$$h : \mathbb{R} \to \mathbb{R}\, \text{is a continuous function} \quad \text{and} \quad h(s)s \geq 0 \quad \text{for all}\, s \in \mathbb{R}. \tag{2.4}$$

($\mathbf{H_3}$) Hypotheses on $\boldsymbol{\gamma}$.

Let γ be a constant satisfying the following condition:

$$0 \leq \gamma < \frac{1}{n-2} \quad \text{if}\, n \geq 3 \quad \text{and} \quad \gamma \geq 0 \quad \text{if}\, n = 1, 2. \tag{2.5}$$