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Inverse eigenvalue problem for normal J-hamiltonian matrices

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ABSTRACT

A complex square matrix A is called J-hamiltonian if AJ is hermitian where J is a normal real matrix such that $J^2 = -I_n$. In this paper we solve the problem of finding J-hamiltonian normal solutions for the inverse eigenvalue problem.

1. Introduction

Inverse eigenvalue problems arise as important tools in several research subjects, including structural design, parameter identification and modeling [1–3], etc. The main goal of the inverse eigenvalue problem is to construct a matrix A with a determined structure and a specified spectrum. In the literature, this kind of problems has been studied under certain constraints on A. For instance, the case when A is hermitian reflexive or anti-reflexive with respect to a tripotent hermitian matrix was analyzed in [4]. Subsequently, that problem was generalized to matrices that are hermitian reflexive with respect to a normal $\{k+1\}$-potent matrix [5]. By using hamiltonian matrices, in [6] Bai solved the inverse eigenvalue problem for hermitian and generalized skew-hamiltonian matrices.

It is remarkable that hamiltonian matrices play an important role in several engineering areas such as optimal quadratic linear control [7,8], H_∞ optimization [9] and the solution of Riccati algebraic equations [10], among others.

The symbols M^* and $M^\dagger$ will denote the conjugate transpose and the Moore–Penrose inverse of a matrix M, respectively. As is standard, I_n will stand for the $n \times n$ identity matrix. We remind the reader that for a

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given complex rectangular matrix $M \in \mathbb{C}^{m\times n}$, its Moore–Penrose inverse is the unique matrix $M^\dagger \in \mathbb{C}^{n\times m}$ that satisfies $MM^\dagger M = M$, $M^\dagger M M^\dagger = M^\dagger$, $(MM^\dagger)^* = MM^\dagger$ and $(M^\dagger M)^* = M^\dagger M$. This matrix always exists [11]. We also need the following notation for both specified orthogonal projectors: $W_M^{(l)} = I_n - M^\dagger M$ and $W_M^{(r)} = I_m - MM^\dagger$.

It is well known that a matrix $A \in \mathbb{C}^{2k\times 2k}$ is called hamiltonian if it satisfies $(AJ)^* = AJ$ for

$$J = \begin{bmatrix} 0 & I_k \\ -I_k & 0 \end{bmatrix}.$$

We extend this concept by considering the following matrices.

Definition 1. Let $J \in \mathbb{R}^{n\times n}$ be a normal matrix such that $J^2 = -I_n$. A matrix $A \in \mathbb{C}^{n\times n}$ is called J-hamiltonian if $(AJ)^* = AJ$.

From now on, we will consider a fixed normal matrix $J \in \mathbb{R}^{n\times n}$ such that $J^2 = -I_n$. It is clear that $n = 2k$ for some positive integer k. For a given matrix $X \in \mathbb{C}^{n\times m}$ and a given diagonal matrix $D \in \mathbb{C}^{m\times m}$, we are looking for solutions of the matrix equation

$$AX = XD \tag{1}$$

where the unknown $A \in \mathbb{C}^{n\times n}$ must be normal and J-hamiltonian.

2. Inverse eigenvalue problem

2.1. General expression for matrices A

Let $J \in \mathbb{R}^{n\times n}$ be a normal matrix satisfying $J^2 = -I_n$. It is easy to see that J is skew-hermitian and its spectrum is included in $\{-i, i\}$ where both eigenvalues i and $-i$ have the same multiplicity, $k = n/2$. Then, there exists a unitary matrix $U \in \mathbb{C}^{n\times n}$ such that

$$J = U \begin{bmatrix} iI_k & 0 \\ 0 & -iI_k \end{bmatrix} U^*. \tag{2}$$

Using block matrices, we can analyze the structure of matrices A as follows. We partition

$$U^*AU = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} \tag{3}$$

according to the partition of J. From (2) and (3), equality $(AJ)^* = AJ$ yields

$$U \begin{bmatrix} -iA_{11}^* & -iA_{21}^* \\ iA_{12}^* & iA_{22}^* \end{bmatrix} U^* = U \begin{bmatrix} iA_{11} & -iA_{12} \\ iA_{21} & -iA_{22} \end{bmatrix} U^*$$

from where we deduce

$$A_{11}^* = -A_{11}, \qquad A_{22}^* = -A_{22}, \qquad A_{21}^* = A_{12}. \tag{4}$$

Since A must be normal, using expressions (4) we get that

$$AA^* = U \begin{bmatrix} -A_{11}^2 + A_{12}A_{12}^* & A_{11}A_{12} - A_{12}A_{22} \\ -A_{12}^*A_{11} + A_{22}A_{12}^* & A_{12}^*A_{12} - A_{22}^2 \end{bmatrix} U^*$$

and

$$A^*A = U \begin{bmatrix} -A_{11}^2 + A_{12}A_{12}^* & -A_{11}A_{12} + A_{12}A_{22} \\ A_{12}^*A_{11} - A_{22}A_{12}^* & A_{12}^*A_{12} - A_{22}^2 \end{bmatrix} U^*$$

imply $A_{11}A_{12} = A_{12}A_{22}$. We have obtained the following result.