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An extension of estimation of domain of attraction for fractional order linear system subject to saturation control

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ABSTRACT

This paper employs the Lyapunov direct method for the stability analysis of fractional order linear systems subject to input saturation. A new stability condition based on saturation function is adopted for estimating the domain of attraction via ellipsoid approach. To further improve this estimation, the auxiliary feedback is also supported by the concept of stability region. The advantages of the proposed method are twofold: (1) it is straightforward to handle the problem both in analysis and design because of using Lyapunov method, (2) the estimation leads to less conservative results. A numerical example illustrates the feasibility of the proposed method.

1. Introduction

Saturation is a ubiquitous phenomenon in physical systems that plays an important role in mathematics and engineering. During the last decades this topic was studied in the scope of Integer Order (IO) systems [1,2]. Hu et al. [1] derived a condition stability for IO linear system subject to actuator saturation in terms of an auxiliary feedback using the ellipsoid approach. However, it is still an open problem of Fractional Order (FO) systems. In spite of the interest in FO dynamical system in modeling and control [3–6], only a few papers were devoted to saturation nonlinearity [7,8]. Lim et al. [7] obtained the sufficient stability based on the solution of the fractional linear equation. They adopted the Gronwall–Bellman lemma and the property of sector bounded saturation in the general case with $0 < \alpha < 2$, where α represents the fractional order. In [8], the stability of FO saturation system is addressed by means of a Lyapunov function using Riemann–Liouville definition. In [9] and references therein, it has been shown that the fractional derivative of Lyapunov function $({}_0^C D_t^\alpha V)$ is a finite series and that there exists a boundedness on ${}_0^C D_t^\alpha V$.

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The contribution of this paper is to analyze the stability of FO linear system with saturation nonlinearity via the relation between the Riemann–Liouville and the Caputo definitions. The Lyapunov direct method is employed where the estimated region of attraction is obtained through the ellipsoid approach based on the boundedness of ${}_0^C D_t^{\alpha} V$. An auxiliary feedback is also utilized to improve the estimation of domain of attraction. Having these idea in mind the paper is organized as follows. Section 2 presents the fundamental concepts. Section 3 describes the problem and formulates the stability analysis. Section 4 estimates the domain of attraction by using auxiliary feedback. Section 5 presents simulation results. Finally, Section 6 outlines the main conclusions.

2. Fundamental concepts

There are several definitions of FO derivatives being well-known the Riemann–Liouville and Caputo formulations [10]. The physical interpretations of fractional derivative are given in [11]. The operators ${}^C D_t^{\alpha}$ and ${}^R D_t^{\alpha}$ denote the Caputo and Riemann–Liouville fractional derivatives, respectively.

Remark 1 (*[12]*)**.** There is the following relation between ${}^C D_t^{\alpha}$ and ${}^R D_t^{\alpha}$.

$$ {}^R D_t^{\alpha} f(t) = {}^C D_t^{\alpha} f(t) + \sum_{k=1}^{m} \frac{t^{k-\alpha}}{\Gamma(k-\alpha+1)} f^k(0). \tag{1} $$

Remark 2 (*[12]*)**.** Using Riemann–Liouville definition, FO derivative of positive constant $a > 0$ is

$$ {}^R D_t^{\alpha}(a) = \frac{a t^{-\alpha}}{\Gamma(1-\alpha)}. \tag{2} $$

In the rest of the paper, we refer to x instead of $x(t)$ to simplify the notation.

Remark 3 (*[9]*)**.** According to Leibniz's rule of differentiation in FO system, the αth order time derivative of $h(x) = x^T x$ can be extended as

$$ D_t^{\alpha} h(x) = (D_t^{\alpha} x)^T x + x^T (D_t^{\alpha} x) + 2\gamma, \tag{3} $$

where

$$ \gamma = \sum_{k=0}^{\infty} \frac{\Gamma(1+\alpha)[D_t^k x]^T [D_t^{\alpha-k} x]}{\Gamma(1+k)\Gamma(1-k+\alpha)}. \tag{4} $$

From [9], γ is bounded as follows.

$$ \|\gamma\| \le \sigma \|x\|, \quad \sigma > 0. \tag{5} $$

Consider the following Lyapunov function:

$$ V = x^T P x \tag{6} $$

where P is a positive definite matrix. According to Remarks 1 and 3, we can easily conclude

$$ {}^C D_t^{\alpha} V = [{}^R D_t^{\alpha} x]^T P x + x^T P [{}^R D_t^{\alpha} x] - {}^R D_t^{\alpha}[x(0)^T P x(0)] + \gamma, \tag{7} $$

where

$$ \|\gamma\| \le \beta \|x\|, \quad \beta > 0. \tag{8} $$

with $\beta = p \times \sigma$ and p is maximum eigenvalue of P.