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Global stabilization of linear time-varying delay systems with bounded controls

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ABSTRACT

The objective of the present paper is to give an explicit solution to the global stabilization problem for linear time-varying delay systems with bounded control. Lyapunov function method with LMI techniques are proposed in order to derive novel sufficient conditions for designing stabilizing feedback control in terms of LMIs. The proposed result is illustrated through a numerical example.

1. Introduction

Global stabilization problem (GSP) of linear systems with bounded control, which concerns the existence of a bounded feedback controller globally stabilizing the closed-loop system, is one of the important issues in control theory and applications [1–5]. The control constraints automatically impose limitations on our ability to steer the dynamics of the control system and usually arise due to the physical limitation of control actuators such as pumps or valves. It is well established that neglecting these constraints while designing controllers can lead to significant performance deterioration and even closed-loop instability. One of the key limitations imposed by constraints is that on the set of initial states that can be steered to the origin with the available control action. An effective control policy, that takes constraints into account, need provide, not only the stabilizing feedback control law, but also an explicit characterization of the set of initial conditions, starting from where closed-loop stability is guaranteed. This realization, together with the prevalence of hard constraints in control applications, has consequently fostered a large and growing body of research work on control of systems subject to input constraints. Examples include results on constrained optimal quadratic control [6,7], model predictive control [8] and feedback stabilization [9–12]. The GSP for linear systems without delays has been solved in existing literature, whereas a solution to the linear systems with time delay is not known to our knowledge. To the best of our knowledge, there have been few research work about the GSP of linear systems with bounded control, and there is no result so far about the global stabilization of linear systems with both the delay and bounded control. Motivated by these considerations, we have developed the results in [12] by considering bounded control input and time-varying delay, and designing a new class of nonlinear stabilizing feedbacks. By constructing a simple set of Lyapunov functionals, some delay-dependent conditions for designing stabilizing feedback control are obtained in terms of linear matrix inequalities (LMIs), [13].

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The rest of the paper is organized as follows. Section 2 introduces the problem to be treated and some preliminaries. We present sufficient conditions for global stabilization of linear time-varying the system with bounded control and a numerical example in Section 3. Some conclusions are drawn in Section 4.

2. Preliminaries

The following notations will be used throughout this paper. $\mathbb{R}^+$ denotes the set of all nonnegative real numbers; $\mathbb{R}^n$ denotes the n-dimensional space; $\mathbb{R}^{n\times m}$ denotes the space of all matrices of $(n \times m)$-dimension; A^T denotes the transpose of A; a matrix A is symmetric if $A = A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A) = \max\{Re(\lambda) : \lambda \in \lambda(A)\}$; $\lambda_{\min}(A) = \min\{Re(\lambda) : \lambda \in \lambda(A)\}$; $C([-\tau, 0], \mathbb{R}^n)$ denotes the set of all $\mathbb{R}^n$-valued continuous functions on $[-\tau, 0]$; The symmetric terms in a matrix are denoted by $*$. Matrix A is positive definite $(A > 0)$ if $(Ax, x) > 0$ for all $x \neq 0$. The segment of the trajectory $x(t)$ is denoted by $x_t = \{x(t+s) : s \in [-\tau, 0]\}$ with the norm $\|x_t\| = \sup_{s\in[-\tau,0]} \|x(t+s)\|$.

Consider the following linear control time-varying delay system with bounded control

$$\begin{aligned}&\dot{x}(t) = Ax(t) + A_1x(t-h(t)) + Bu(t), \quad t \geq 0,\\&x(t) = \phi(t), \quad t \in [-h, 0]\end{aligned} \tag{2.1}$$

where $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathbb{R}^m$ is the control, $A, A_1 \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ are given constant matrices. We assume that the matrix A has full column rank. The initial function $\phi(t) \in C([-h, 0], \mathbb{R}^n)$ and $h(t) : \mathbb{R}^+ \to \mathbb{R}^+$ is a continuous function satisfying

$$0 \leq h_1 \leq h(t) \leq h_2, \quad \forall t \geq 0. \tag{2.2}$$

The control $u(.)$ is a continuous function satisfying the following condition

$$\|u(t)\| \leq r, \quad \forall t \geq 0. \tag{2.3}$$

Definition 2.1. The control system (2.1) is globally stabilizable if there is a feedback control $u(t) = k(x(t))$ satisfying the constraint (2.3) such that the resulting closed-loop system:

$$\dot{x}(t) = Ax(t) + A_1x(t-h(t)) + Bk(x(t)), \quad t \in R^+$$

is globally asymptotically stable.

Proposition 2.1 (*Schur Complement Lemma [14]*). *Given constant matrices X, Y, Z with appropriate dimensions satisfying $X = X^T$ and $Y = Y^T > 0$, then $X + Z^TY^{-1}Z < 0$ if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y \end{pmatrix} < 0.$$

Proposition 2.2 (*Integral Inequality [15]*). *For any constant matrix $Z = Z^T > 0$ and a number $h > 0$ such that the following integrations are well defined, we have*

$$h\int_{t-h}^{t} x(s)^TZx(s)ds \geq \left(\int_{t-h}^{t} x(s)ds\right)^T Z \left(\int_{t-h}^{t} x(s)ds\right), \quad t \geq 0.$$

3. Stabilizability conditions

In this section, we study the global stabilization of system (2.1) and give sufficient conditions for designing feedback controllers in terms of LMIs. Firstly, we prove the existence of solutions of the closed-loop system using nonlinear state feedback control:

$$u(t) = -\frac{r}{1 + \|B^TPx(t)\|}B^TPx(t), \quad t \geq 0. \tag{3.1}$$

Lemma 3.1. *By the state feedback control (3.1), the nonlinear closed-loop system*

$$\dot{x}(t) = Ax(t) + A_1x(t-h(t)) - r\frac{BB^TPx(t)}{1 + \|B^TPx(t)\|}, \quad t \in \mathbb{R}^+,$$

has a unique solution on $\mathbb{R}^+$.