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# On real-valued oscillations of a bipendulum

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ABSTRACT

Theoretical and computational aspects of special case of logistical-routing problem are considered. Fluctuation of two particles on a grid connected by a channel also is considered. Velocity rate and sufficient conditions of system self-regulation are obtained.



## 1. Formulation of problem

Consider a dynamical system with discrete time $T = 0, 1, \ldots$. There are *two vertices* $V_0, V_1$ and *two particles* $P_0, P_1$. Each particle is at one of two vertices at each time instant, Fig. 1.

Suppose the following binary representations of two numbers $a_0, a_1, 0 \le a_0, a_1 < 1$,

$$a_0 = 0, \quad a_{01}a_{02}\ldots a_{0k}\ldots; \qquad a_1 = 0, \quad a_{11}a_{12}\ldots a_{1k}\ldots, \tag{1}$$

determine the plan of particles relocations. The particle $P_i$ must be located, at successive time instants, in the vertex $V_{a_{iT}}$ in accordance with the plan of this particle, $i = 0, 1, T = 1, 2, \ldots$. If the particle $P_i$ is in the vertex $V_{a_{iT}}$ at the instant $T$, then, at the instant $T + 1, T = 0, 1, 2, \ldots$, this particle will be in the cell $V_{a_{i(T+1)}}$ except the following cases.

**Scenario A.** *If particles are at different vertices at the same instant $T$, and each particle must come in accordance with its plan to the other vertex, then a competition, or a conflict, takes place.*

In the case of a competition, only one of two competing particles changes its state, and the other particle does not change its state, and will change its state at the next step, in accordance with the conflict resolution rule. Thus the implementation of the plan is delayed at least for a step. Assume that some initial conditions are given. These conditions determine the location of particles at the instant $T = 0$. Assume that $\Delta a_{ik} = a_{i,k+1} - a_{ik},\ i = 0, 1,\ k = 1, 2, \ldots$.

**Proposition 1.** *In the case of the scenario A, the competition takes place at the instant $T$ where $T$ is the minimum positive instant such that $\Delta a_{0T}\Delta a_{1T} = -1$.*

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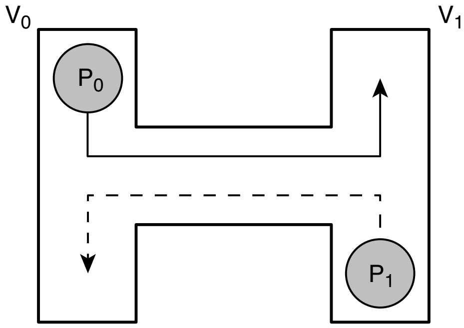


**Fig. 1.** Movement in the opposite directions.


**Fig. 2.** Turing tapes.

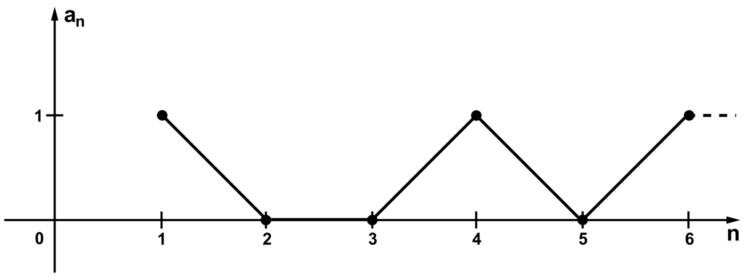


**Fig. 3.** Functional interpretation.

## 2. Geometrical interpretation

### *2.1. Particles and channels*

The system that we consider is the simplest version of the generalized version of the transport logistic problem which has been formulated in [1] at the Conference CMMSE — 2014. We shall give a geometrical interpretation of the system. We assume that two vertices can contain simultaneously two particles, and the vertices are connected by a channel, and the width of the channel corresponds to the width of a particle, Fig. 1. Particles must move or not move during at each step in accordance with plans. If one of the particle moves and the other particle does not move, then no conflicts take place. A competition takes place if two particles attempt to move in opposite directions in the case of any length of the channel. Only the particle, winning competition moves.

### *2.2. Turing tapes*

Two semi-infinite tapes with cells, containing the logistic of particles, move to the left at a gradation per a unit of discrete time relatively to a given level if there are no competitions. Otherwise only the tape of the winning particle moves, and the other tape does not move. Consider a stereotyped pattern containing two rows and two columns. Two rows correspond to particles. Two columns correspond to the *present and the future*. The case of the competition corresponds to unit matrix or skew unit matrix for the scenario A (see Figs. 2 and 3).

### *2.3. Functional interpretation*

We shall give definition of the function $b_i(T)$. Assume that

$$b_i(T) = j, \quad i = 0, 1, \; T = 1, 2, \ldots,$$

if, for the given realization of process, the particle $P_i$ is in the vertex $V_j$, $i, j = 0, 1$, at the instant $T$. Thus the sequences $a_i = a_{i,T}$ give a plan of the relocation of the particles $V_i$, $i = 0, 1$, and the sequences $b_i(T) = b_{i,T}$ give the plan implementation.