Applied Mathematics Letters 45 (2015) 69–75

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

A multiphase first order model for non-equilibrium sand erosion, transport and sedimentation

L. Preziosi [a,d,*], D. Fransos [b,d], L. Bruno [c,d]

[a] Department of Mathematical Sciences – Politecnico di Torino, Corso degli Abruzzi 24, I-10129, Torino, Italy

[b] Optiflow Company, 160, Chemin de la Madrague-Ville, F-13015, Marseille, France

[c] Department of Architecture and Design – Politecnico di Torino, Viale Mattioli 39, I-10125, Torino, Italy

[d] Windblown Sand Modeling and Mitigation joint research group

ARTICLE INFO

Article history:
Received 29 December 2014
Received in revised form 20 January 2015
Accepted 20 January 2015
Available online 4 February 2015

Keywords:
Multiphase model
Sand transport
Erosion
Sedimentation

ABSTRACT

Three phenomena are involved in sand movement: erosion, wind transport, and sedimentation. This paper presents a comprehensive easy-to-use multiphase model that include all three aspects with a particular attention to situations in which erosion due to wind shear and sedimentation due to gravity are not in equilibrium. The interest is related to the fact that these are the situations leading to a change of profile of the sand bed.

1. Introduction

When the shear stress exerted by wind on a sandy surface is sufficiently strong, sand grains are lifted from the sand bed and are transported by wind to sediment downstream. The raising sand grains follow a ballistic trajectory influenced by drag and gravity, eventually impacting again on the surface and inducing new particles to detach from the surface. This phenomenon, known as saltation, generates a layer close to the sand bed with a typical maximum height of 10–20 cm. Saltation is the main reason of erosion of sandy surfaces and together with the consequent sedimentation of sand particles it is the main reason of dune motion and accumulation of sand in specific regions where recirculation occurs.

The engineering interest in understanding and simulating the dynamics of windblown sand, e.g. dune fields of loose sand [1], is dictated by their interaction with a number of human infrastructures in arid environments, such as roads and railways, pipelines, industrial facilities, farmlands, towns and buildings, as shown in Fig. 1. From a phenomenological point of view, moving intruder sand dunes, soil erosion and/or sand contamination can be comprehensively ascribed to non-equilibrium conditions, where the two processes, erosion and sedimentation, do not balance, leading to the erosion or deposition of sand on the soil and eventually to the evolution of the sandy surface. In other terms, such non-equilibrium situations are the most interesting cases from the applicative point of view.

Bagnold [2] was the first who studied sand erosion and postulated a relation for the sand flux, determining the importance of wind speed and of the related shear stress on the sand surface. Later authors [3–10] introduced several corrections to

* Corresponding author at: Department of Mathematical Sciences – Politecnico di Torino, Corso degli Abruzzi 24, I-10129, Torino, Italy. Tel.: +39 011 090 7555; fax: +39 011 090 7599.

E-mail address: luigi.preziosi@polito.it (L. Preziosi).

URL: http://www.polito.it/wsmm (L. Preziosi).

http://dx.doi.org/10.1016/j.aml.2015.01.011
0893-9659/

Fig. 1. Windblown sand interaction with anthropic activities: megadunes surrounding a road and farmlands in Dunhuang, Gansu Province, PRC, photo: I.A. Inman, 2007 (a), linear dunes encroaching Nouakchott, capital of Mauritania. Landsat 1565-10032-6, 1974 (b), loose sand covering the Aus to Lüderitz railway Line, Namibia photo: K. Dierks, 2003 (c).

Bagnold's rule, but all the models have in common the observation that a sand grain is ejected from a sand bed if and only if the shear stress at the surface is larger than a threshold value.

Sauermann et al. [9] observed that saltation reaches a steady state after a transitory phase of 2 s. After this period the trajectories are statistically equivalent for the ensemble of grains. This phenomenon happens because the new ejected particles increase the sand concentration in the saltation layer and this reduces the speed of saltating grains. So, a steady state is reached when all particles are ejected with the same velocity (see also [10–13]). A nice mathematical models of the saltation phenomenon is proposed by Herrmann and Sauermann [14] who studied the dynamics of the surface of a dry granular bed dividing the sand bed into a non-moving time-dependent region providing sand mass and another time-dependent region above it in which sand particles can move transported by the wind. They proposed a model averaged over the vertical coordinate, presenting a free boundary.

Ji et al. [5] coupled a k–ϵ model with a multiphase approach in which the slip of the dispersed phase is modelled by an algebraic model. Similar turbulent one-dimensional models are proposed in [9,15–17], however without a multiphase coupling. Kang and coworkers [18–20] instead couple a multiphase model for the fluid flow with a particle method for the sand grains. A similar coupling was also used in [11,21,22] where however the wind flow was computed *independently* from the presence of sand particles via a suitable turbulence model, typically the k–ϵ model.

Sedimentation has also been widely studied in the literature starting from several applications mainly in environmental and chemical engineering. One of the most important component in this phenomenon is the drag force experienced by the sedimenting particles that has driven a lot of attention by many authors as well reviewed in [23].

Differently from previous papers, here we will propose a comprehensive multiphase model for the entire process including sand erosion, wind transport, and sedimentation, that working also in non-equilibrium conditions is able to deal with the development of the stationary saltation layer starting from generic initial and boundary conditions and in particular from clear air and over-saturated situations. In order to do that, we develop a so-called first order model (in time) of sand erosion, transport and deposition, that can be easily tuned using experimental test cases. The resulting advection–diffusion equation for the suspended phase can then be coupled with a k–ω model describing the turbulent fluid flow. The mathematical model can then be solved with the aid of the fundamental erosion/deposition boundary condition at the sand bed, that depends on the shear stress.

The plan of the paper is then the following. After this introduction, Section 2 presents the mathematical model mainly focusing on the advective phenomena, on the microscopic dynamics related to the collision between sand grains, and on the erosion boundary condition. The result of some numerical simulations focusing on how the stationary condition is reached when wind blows over a heterogeneous sand bed are reported in Section 3.

2. The erosion/transport/deposition model

We consider the flow of sand as a multiphase system composed of sand grains in air. Single sand grains have a density $\hat{\rho}_s$ and float in air with a volume ratio ϕ_s (typically well below 1%), so that the partial density of sand in air is $\rho_s = \hat{\rho}_s\phi_s$. Saturation obviously implies that $\phi_f = 1 - \phi_s$ where ϕ_f is the volume ratio of air. The mixture of air and sand grains is flowing on a sandy surface having a close packing volume ratio $\bar{\phi}_s$.

Because wind flow is in a turbulent regime the fluid phase is modelled by the Reynolds-averaged Navier–Stokes equations (RANS) equations. More precisely, a k–ω turbulence model is selected to provide the closure [24]

$$\begin{cases} \nabla\cdot\mathbf{v}_f = 0 \\ \rho_f\left(\dfrac{\partial\mathbf{v}_f}{\partial t} + \mathbf{v}_f\cdot\nabla\mathbf{v}_f\right) = -\nabla p + \nabla\cdot[\rho_f(\nu_a+\nu_t)\nabla\mathbf{v}_f] \\ \dfrac{\partial k}{\partial t} + \nabla\cdot(k\mathbf{v}_f) = \nabla\cdot[(\nu_a+\nu_t)\nabla k] + P_k - \gamma\omega k \\ \dfrac{\partial\omega}{\partial t} + \nabla\cdot(\omega\mathbf{v}_f) = \nabla\cdot[(\nu_a+\nu_t)\nabla\omega] + P_\omega - C_\omega\omega^2 \end{cases} \tag{1}$$