Applied Mathematics Letters 42 (2015) 22–29

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

The pth moment exponential ultimate boundedness of impulsive stochastic differential systems[☆]

Liguang Xu[a,b,*], Shuzhi Sam Ge[b]

[a] Department of Applied Mathematics, Zhejiang University of Technology, Hangzhou 310023, PR China

[b] Department of Electrical and Computer Engineering, National University of Singapore, Singapore 117576, Singapore

ARTICLE INFO

Article history:
Received 12 September 2014
Received in revised form 27 October 2014
Accepted 27 October 2014
Available online 11 November 2014

Keywords:
Stochastic
Impulsive
Exponential stability
Exponential ultimate boundedness

ABSTRACT

In this paper, we investigate the global pth moment exponential ultimate boundedness of impulsive stochastic differential systems. Using Lyapunov functions and algebraic inequality techniques, some sufficient conditions ensuring the global pth moment exponential ultimate boundedness of the systems are obtained. It is shown that an unstable stochastic differential system can be successfully stabilized by impulses, even more, an unbounded stochastic differential system can be made into a bounded system under a proper impulsive control law. An example is also given to explain our results.

1. Introduction

Stochastic differential systems are an important class of dynamical systems whose evolution in time is governed by random forces. In the past few decades, stochastic differential systems have attracted considerable research interests due to their potential applications in control engineering, mechanical, electrical, neural networks, economic sciences and physical sciences. A large amount of important papers have appeared on the theory of stochastic differential systems; see [1–16] and the references therein.

In addition to the stochastic effects, as is well known, impulsive effects are common phenomena which can be found in a wide variety of real systems such as medicine and biology, economics, mechanics, electronics and telecommunications, etc. As a result, it is natural and necessary to consider impulsive effects to the stochastic differential systems. Recently, the interest of researchers in impulsive stochastic differential systems has increased phenomenally and many interesting results on impulsive stochastic differential systems have been reported [17–31]. These results cover stability, attractivity, invariance, existence and uniqueness of impulsive stochastic differential systems.

On the other hand, boundedness is another important asymptotic property of dynamical systems, which plays an important role in investigating stability, invariant and attracting properties, the uniqueness of equilibrium, the existence of periodic solution and so on. However, to the best of our knowledge, at present there are few reports on the boundedness of impulsive stochastic differential systems. Motivated by this lack, the present paper is focused on the boundedness of impulsive stochastic differential systems. Using Lyapunov functions and algebraic inequality techniques, some sufficient conditions ensuring the global pth moment exponential ultimate boundedness of the systems are obtained. The results

☆ The work is supported by National Natural Science Foundation of China (Grant No. 11101367) and the China Scholarship Council (Grant No. 201208330001).

* Correspondence to: Department of Applied Mathematics, Zhejiang University of Technology, Hangzhou 310023, PR China.
E-mail address: xlg132@126.com (L. Xu).

http://dx.doi.org/10.1016/j.aml.2014.10.018
0893-9659/

show that an unstable stochastic differential system can be successfully stabilized by impulses, even more, an unbounded stochastic differential system can be made into a bounded system under a proper impulsive control law.

The rest of this paper is organized as follows. In Section 2, some notations and definitions are given. In Section 3, some theorems on pth moment exponential ultimate boundedness for impulsive stochastic differential systems are established. In Section 4, an example is given to illustrate our results. Finally, a conclusion is given in Section 5.

2. Preliminaries

Throughout this paper, we use the following notations. Let $\mathbb{R}^n$ ($\mathbb{R}^n_+$) be the space of n-dimensional (nonnegative) real column vectors and $\mathbb{R}^{m\times n}$ be the set of $m \times n$ real matrices. Let $\|\cdot\|$ denote the Euclidean norm in $\mathbb{R}^n$, $\mathbb{N} = \{1, 2, 3, \ldots\}$, $\mathbb{R}_+ = [0, \infty)$ and $\mathbb{R}_{t_0} = [t_0, \infty)$. $\mathscr{C}^{1,2}(\mathbb{R}_{t_0} \times \mathbb{R}^n, \mathbb{R}_+)$ denotes the family of all nonnegative functions from $\mathbb{R}_{t_0} \times \mathbb{R}^n$ to $\mathbb{R}_+$, which are once continuously differentiable in the first variable and twice in the second one. $\mathscr{L}$ denotes the well-known $\mathscr{L}$-operator given by Itô's formula (for more details about $\mathscr{L}$-operator, cf. [7,9]). Denote by $\lambda_{\max}(\cdot)$ the maximum eigenvalue of a matrix. $\mathbb{E}(\cdot)$ means the expectation of a stochastic process.

Consider the following impulsive stochastic differential system:

$$\begin{cases} dx(t) = f(t, x(t)) + g(t, x(t))d\omega(t), & t \neq t_k, t \geq t_0, \\ x(t_k^+) = x(t_k) + I_k(t_k, x(t_k)), & k \in \mathbb{N}, \\ x(t_0) = x_0, \end{cases} \tag{1}$$

where $\omega(t) = (\omega_1(t), \omega_2(t), \ldots, \omega_m(t))^\top$ is an m-dimensional Brownian motion defined on the classical complete probability space $(\Omega, \mathscr{F}, \mathbb{P})$ with a natural filtration $\{\mathscr{F}_t\}_{t\geq t_0}$, in which Ω is a sample space, $\mathscr{F}$ is a σ-field, and $\mathbb{P}$ is a probability measure. $f : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^{n\times m}$, $I_k : \mathbb{R}_+ \times \mathbb{R}^n \to \mathbb{R}^n$, and the fixed impulsive moments $t_k (k \in \mathbb{N})$ satisfy $t_1 < t_2 < \cdots < t_k < \cdots$ and $\lim_{k\to\infty} t_k = \infty$.

Throughout this paper, we assume that for any $x_0 \in \mathbb{R}^n$, there exists at least one solution of system (1), which is denoted by $x(t, t_0, x_0)$ (simply $x(t)$ if no confusion should occur). One may refer to [20,21,26] for the result on the existence and uniqueness of the solutions of impulsive stochastic differential systems.

Definition 2.1. System (1) is said to be globally pth moment exponentially ultimately bounded if there exist constants $\lambda > 0$, $K > 0$ and $M \geq 0$ such that for any solution with the initial condition $x_0 \in \mathbb{R}^n$,

$$\mathbb{E}\|x(t)\|^p \leq K\mathbb{E}\|x_0\|^p e^{-\lambda(t-t_0)} + M, \quad p > 0,\ t \geq t_0, \tag{2}$$

where M is said to be an ultimate bound of system (1). When $p = 2$, it is usually called globally exponentially ultimately bounded in mean square.

Definition 2.2. System (1) is said to be globally pth moment exponentially stable if there exist positive constants λ and K such that for any solution with the initial condition $x_0 \in \mathbb{R}^n$,

$$\mathbb{E}\|x(t)\|^p \leq K\mathbb{E}\|x_0\|^p e^{-\lambda(t-t_0)}, \quad p > 0,\ t \geq t_0. \tag{3}$$

When $p = 2$, it is usually called globally exponentially stable in mean square.

3. Main results

In this section, we will apply the Lyapunov functions and algebraic inequality techniques to investigate the pth moment exponential ultimate boundedness of system (1).

Theorem 3.1. *Assume that there exist a function $V(t, x) \in \mathscr{C}^{1,2}(\mathbb{R}_{t_0} \times \mathbb{R}^n, \mathbb{R}_+)$ and several constants $\Upsilon \geq 0$, $\xi > 1$, $\beta_k > 0$, $c_1 > 0$, $c_2 > 0$ and α such that*

(i) *for all $(t, x) \in \mathbb{R}_{t_0} \times \mathbb{R}^n$,*

$$c_1\|x\|^p \leq V(t, x) \leq c_2\|x\|^p; \tag{4}$$

(ii) *for all $k \in \mathbb{N}$ and $x \in \mathbb{R}^n$,*

$$V(t_k^+, x(t_k) + I_k(t_k, x(t_k))) \leq \beta_k V(t_k, x(t_k)); \tag{5}$$

(iii) *for all $t \geq t_0$, $t \neq t_k$, $k \in \mathbb{N}$, and $x \in \mathbb{R}^n$,*

$$\mathscr{L}V(t, x(t)) \leq \alpha V(t, x(t)) + \Upsilon; \tag{6}$$

(iv) $0 < T_k = t_k - t_{k-1} < \infty$, $k \in \mathbb{N}$,

$$\ln \xi\beta_k + \alpha T_k < 0. \tag{7}$$