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Dynamics and optimal control in a spatially structured economic growth model with pollution diffusion and environmental taxation

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ABSTRACT

In the first part of this paper we present a spatially structured dynamic economic growth model which takes into account the level of pollution and a possible taxation based on the amount of produced pollution. In the second part we analyze an optimal harvesting control problem with an objective function composed of three terms, namely the intertemporal utility of the decision maker, the space–time average of the level of pollution in the habitat, and the disutility due to the imposition of taxation.

1. Introduction

Since the seminal paper by Grossman and Kruger [1] there has been considerable academic interest in the relationship between economic development and environmental pollution. In view of the recent policy developments, resolving this issue seems of particular importance (see e.g. [2]). Of particular interest is the report [3] (see also references therein) recently issued by the International Monetary Fund on the role of taxation for controlling pollution.

In this paper we move from our proposed geographical economic growth model [4], in which we have analyzed the impact of both capital and pollution diffusion on the economic and environmental joint dynamics, and consider an additional optimal control problem which takes into account a possible taxation based on the amount of produced pollution.

Standard macroeconomics and environmental economics have been two completely independent research areas for a long time. Only recently some works tend to develop a global theory combining these two branches of literature (see Brock and Taylor [5]). The inclusion of a spatial dimension in economic analysis has recently increased relevance and interest. The first studies in economic geography go back to Beckman [6] and Puu [7], who study regional problems based simply on flow

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equations. Starting from these works, a new economic geography arises adopting general equilibrium models to analyze the peculiarities of local and global markets, and the mobility of production factors (Krugman [8]; Fujita et al. [9]). More recently, this geographical approach has been introduced in economic growth models to study the implications between accumulation and diffusion of capital on economic dynamics (Brito [10]; Camacho and Zou [11]; Boucekkine et al. [12]; Capasso et al. [13]). The Solow model [14] with a continuous spatial dimension has been extensively studied. Camacho and Zou [11] analyze problems of convergence across regions when capital is mobile, while Brito [10] considers the case in which both capital and labor are mobile. The Ramsey model [15] has been extended to a spatial dimension by Brito [10] and Boucekkine et al. [12], respectively in average and total utilitarianism version. In the literature other contributions which explore the spatial dimension in environmental and resource economics can be found in Brock and Xepapadeas [16,17], Xepapadeas [18], and Athanassoglou and Xepapadeas [19].

In Section 2 we introduce our dynamical model, including a taxation rate depending upon the level of pollution at each spatial location and time, and a penalization of the production function due itself to the level of pollution. We have kept the shape of the production function introduced in [13], so to allow a convex–concave choice.

In Section 3 we analyze the large time behavior of the system with given parameters.

In Section 4 we analyze an optimal harvesting control problem with an objective function composed of three terms; the first term, to be maximized, describing the intertemporal utility of the decision maker; the second term, to be minimized, representing the space–time average of the level of pollution in the habitat; the third term, to be minimized, taking into account the disutility due to the imposition of taxation. The resulting control is a bang–bang one.

2. The model

Let $k(x,t)$ and $p(x,t)$ respectively denote the capital and the pollution stock faced by a representative household located at x at date t, in a habitat $\overline{\Omega}$ (where $\Omega \subset \mathbf{R}^N$ is taken as a nonempty and bounded domain with a smooth boundary), and $t \geq 0$. We also assume that the initial capital and pollution distribution, $k(x,0) = k_0(x)$ and $p(x,0) = p_0(x)$, are known and satisfy

$$k_0,\ p_0 \in L^\infty(\Omega), \quad k_0(x) \geq k_{00} > 0,\ p_0(x) \geq 0 \text{ a.e. } x \in \Omega \tag{1}$$

and there is no capital or pollution flow through the boundary of Ω, namely that the normal derivatives $k_\nu(x,t) = p_\nu(x,t) = 0$ at $x \in \partial\Omega$ and $t \geq 0$. We assume a continuous space structure of both physical capital and pollution, so that the model we are interested in describes the evolution of physical capital and pollution emissions over space and time and it is modeled as follows:

$$\begin{cases} k_t(x,t) = d_1 \Delta k(x,t) + \dfrac{s}{1+p(x,t)^2} f((1-\tau(x,t))k(x,t)) - \delta_1 k(x,t) - c(x,t)k(x,t), & (x,t) \in Q_{0,\infty} \\ p_t(x,t) = d_2 \Delta p(x,t) + \theta \displaystyle\int_\Omega f((1-\tau(x',t))k(x',t))\varphi(x',x)dx' - \delta_2 p(x,t), & (x,t) \in Q_{0,\infty} \end{cases} \tag{2}$$

subject to homogeneous Neumann boundary conditions

$$k_\nu(x,t) = p_\nu(x,t) = 0, \quad (x,t) \in \Sigma_{0,\infty}, \tag{3}$$

and initial conditions

$$k(x,0) = k_0(x), \qquad p(x,0) = p_0(x), \quad x \in \Omega. \tag{4}$$

The control variable $c(x,t) \in [0,L]$ describes the level of consumption per unit of physical capital $k(x,t)$ at the location x at the time t, whilst the control variable $\tau(x,t) \in [0,1]$ represents the level of taxation at the location x and at the time t, and $d_1, d_2, s, \theta, \delta_1, \delta_2, L$ are positive parameters. Here $Q_{a,b} = \Omega \times (a,b)$ and $\Sigma_{a,b} = \partial\Omega \times (a,b)$. We assume that $c \in L^\infty(Q_{0,\infty})$ and the convex–concave production function f (see [20]) is of the following form

$$f(r) = \frac{\alpha_1 r^\gamma}{1+\alpha_2 r^\gamma}, \tag{5}$$

where $\alpha_1 \in (0,+\infty),\ \alpha_2 \in [0,+\infty),\ \gamma \in (0,+\infty)$. The term $(1-\tau(x,t))k(x,t)$ accounts for the amount of the physical capital to be used in the gross domestic production. An increase in the level of taxation produces an abatement of the level of pollution emissions, which we can interpret as a "green" taxation policy. We also assume that the level of taxation and the amount of pollution are not independent but they are related to each other through the relationship

$$\tau(x,t) = \frac{\rho p(x,t)}{\phi + \rho p(x,t)} \tag{6}$$

where ρ and ϕ are positive parameters. Finally, φ is a kernel which satisfies the following hypotheses: $\varphi \in L^\infty(\Omega \times \Omega)$, and $\varphi(x',x) \geq 0$ a.e. $(x',x) \in \Omega \times \Omega$. As a consequence of Eq. (6), an increment in pollution emissions produces an increment of the level of taxation.