Applied Mathematics Letters 39 (2015) 1–6

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Model order reduction for a family of linear systems with applications in parametric and uncertain systems

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ARTICLE INFO

Article history:
Received 25 March 2014
Received in revised form 1 August 2014
Accepted 1 August 2014
Available online 13 August 2014

Keywords:
Parametric model order reduction
$\mathcal{H}_2$ optimal interpolation points
Rational approximation

ABSTRACT

We present an approach which creates a linear reduced order model whose transfer function interpolates at certain given points and approximates at others. We describe how to create the corresponding state space system and explain how this method is used for the simulation of large scale parametric systems or several realizations of an uncertain system. In that case, the interpolation points are the $\mathcal{H}_2$ optimal interpolation points of one particular system. The behavior of the method is illustrated in some numerical examples.

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1. Introduction

The original motivation for this work arouse from parametric model order reduction. It is known that for nonparametric systems [1], the $\mathcal{H}_2$-optimal reduced system of order r is created by Hermite interpolation at the mirror images of the reduced poles. We do give the necessary background to this in Section 2. More details can be found in [2–4]. Given a parametric or uncertain system we are interested in creating a good reduced order model for each parameter in a given domain for several realizations of the uncertain system. Our focus in this paper is not to create a truly parametric or uncertain reduced order system but an algorithm which reduces the system for a given parameter (or realization) fast, having knowledge from another parameter (or realization). Details are discussed in Section 3. This could be useful in speeding up the creation of a truly parametric reduced system. Assuming we know the $\mathcal{H}_2$ optimal reduced order model and the optimal interpolation points at a specific parameter we would like to use this information to create a reduced order model at nearby parameters. We are looking at model order reduction (MOR) from the point of view of rational interpolation and use the barycentric form for that. The reduced order model interpolates the transfer function at given points $\sigma_1, \ldots, \sigma_r$ as well as approximates at certain other points $\xi_1, \ldots, \xi_N$. This idea arose from the quite often encountered situation that we have approximate optimal interpolation points $(\sigma_1, \ldots, \sigma_r)$ together with actual frequency response data of the system $(\xi_1, \ldots, \xi_N)$. We describe how to create the rational approximant and the corresponding state space system in Section 4. We will then describe how this leads to a method (Section 5) and show some first rather academic examples in Section 6.

2. Background

Given a large scale stable single input single output dynamical system

$$\Sigma : \begin{cases} E\dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \tag{1}$$

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http://dx.doi.org/10.1016/j.aml.2014.08.002
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where $E, A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^n$, $C \in \mathbb{R}^{1\times n}$, $D \in \mathbb{R}$ such that the pencil (E, A) has only finite eigenvalues in the left half plane, MOR attempts to find a reduced stable dynamical system:

$$\hat{\Sigma} : \begin{cases} \hat{E}\dot{x} = \hat{A}x + \hat{B}u \\ \hat{y} = \hat{C}x + \hat{D}u \end{cases}$$

where $\hat{E}, \hat{A} \in \mathbb{R}^{r\times r}$, $\hat{B} \in \mathbb{R}^r$, $\hat{C} \in \mathbb{C}^{1\times r}$, $\hat{D} \in \mathbb{R}$ and the matrix pencil $(\hat{E}, \hat{A})$ has eigenvalues only in the open left half plane. Here $r \ll n$ and the map from the input $u \in \mathcal{L}_2(\mathbb{R}^+)$ to the output y of the original system can be well approximated by the map from input to output $\hat{y}$ of the reduced system. In the frequency domain, the input–output behavior is characterized by the transfer function. The transfer function $H = C(sE - A)^{-1}B + D$ of the original function and the transfer function $\hat{H} = \hat{C}(s\hat{E} - \hat{A})^{-1}\hat{B} + \hat{D}$ are complex-valued proper rational functions defined on the complex plane. Looking at the maximal error of the difference between the true output and the reduced output we can bound the $\mathcal{L}_\infty$-norm by the $\mathcal{H}_2$-norm [2]: $\operatorname{ess\,sup}_{t>0} |y(t) - \hat{y}(t)| \le \|H - \hat{H}\|_{\mathcal{H}_2} \|u\|_{\mathcal{L}_2}$, where the $\mathcal{H}_2$-norm is defined as $\|H - \hat{H}\|^2_{\mathcal{H}_2} = \frac{1}{2\pi}\int_{-\infty}^{\infty} |H(\imath\omega) - \hat{H}(\imath\omega)|^2 d\omega$. The reduced order model we want to find minimizes the $\mathcal{H}_2$-error for a given r. Since the difference $H(\imath\omega) - \hat{H}(\imath\omega)$ converges to $D - \hat{D}$ as $\omega \to \infty$ we will take $\hat{D} = D$ in our reduced order modeling. Therefore we can w.l.o.g. assume $D = 0$ and thus concentrate on strictly proper rational functions.

$\mathcal{H}_2$ optimal model order reduction

Given a state space system (1) it is known that the $\mathcal{H}_2$-optimal reduced order system of order r Hermite interpolates at $\sigma_1, \ldots, \sigma_r$, the mirror images of the poles of the reduced order system. Since these are not known a priori we need to compute them by an algorithm. The one we will use is called IRKA [2]. Many extensions and improvements of the basic algorithm exist [4,3,5]. We will however use only the basic form. Given the interpolation points, the reduced order model is typically generated by creating projection matrices V and W such that the reduced order system is given by

$$\hat{E} = W^T EV, \qquad \hat{A} = W^T AV, \qquad \hat{B} = W^T B, \qquad \hat{C} = CV.$$

The projection matrices need to be picked such that $(\sigma I - A)^{-1}B \in \operatorname{Ran}(V)$ and $(\bar{\sigma} I - A^T)^{-1}C^T \in \operatorname{Ran}(W)$ [6], where Ran denotes the range of a matrix. However given $\sigma_k, H(\sigma_k), H'(\sigma_k)$ we can also write down a state space system more directly. This is related to the Loewner framework of reduced order modeling and explained in the following.

Loewner framework

Given frequencies together with the value of the transfer function at those frequencies, a data driven approach to MOR is to create a state space system which interpolates there, see [7,8]. Given interpolation points $(\xi_1, \ldots, \xi_N, \sigma_1, \ldots, \sigma_r)$, and its transfer function values $W = [H(\sigma_1), \ldots, H(\sigma_r)]$ and $V^T = [H(\xi_1), \ldots, H(\xi_N)]$, we can define the Loewner matrices $\mathbb{L}, \sigma\mathbb{L}$ and the symmetric Loewner matrices $\mathbb{L}^s, \sigma\mathbb{L}^s$

$$\begin{aligned} &\mathbb{L}_{ij} = \frac{V_i - W_j}{\xi_i - \sigma_j}, \qquad \sigma\mathbb{L}_{ij} = \frac{\xi_i V_i - \sigma_j W_j}{\xi_i - \sigma_j}, \\ &\mathbb{L}^s_{ij} = \begin{cases} \dfrac{W_i - W_j}{\sigma_i - \sigma_j} & \text{if } i \ne j \\ H'(\sigma_i) & \text{if } i = j \end{cases} \qquad \sigma\mathbb{L}^s_{ij} = \begin{cases} \dfrac{\sigma_i W_i - \sigma_j W_j}{\sigma_i - \sigma_j} & \text{if } i \ne j \\ H(\sigma_i) + \sigma_i H'(\sigma_i) & \text{if } i = j. \end{cases} \end{aligned} \tag{2}$$

If $N = r$ the order r reduced state space system that interpolates is then given by

$$\hat{E} = -\mathbb{L}, \qquad \hat{A} = -\sigma\mathbb{L}, \qquad \hat{B} = V, \qquad \hat{C} = W, \tag{3}$$

or for Hermite interpolation of $\sigma_1, \ldots, \sigma_r$ by $\hat{E} = -\mathbb{L}^s$, $\hat{A} = -\sigma\mathbb{L}^s$, $\hat{B} = W^T$, $\hat{C} = W$.

3. Parametric model order reduction and uncertain systems

Several approaches to create parametric reduced order models can be found in the literature. A good start is a recent review article [9]. They often involve reducing the system via nonparametric methods at individual parameters. This paper shows how we can create a reduced order model for a given parameter rapidly knowing the reduced order model of a nearby parameter. At the nearby parameter our approach creates the reduced system by only knowing frequency response data at certain points. These points are the $\mathcal{H}_2$ optimal interpolation points of the original system and certain other points (picked depending on the application). At the $\mathcal{H}_2$ optimal points the reduced order system will interpolate and at the others, it is a good approximation. This idea could be particularly interesting for uncertain system. We reduce one realization of this system. With this information we can then create a reduced order model of a new realization simply and fast.

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