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Convergence rates of trinomial tree methods for option pricing under regime-switching models

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ABSTRACT

Recently trinomial tree methods have been developed to option pricing under regime-switching models. Although these novel trinomial tree methods are shown to be accurate via numerical examples, it needs to give a rigorous proof of the accuracy which can theoretically guarantee the reliability of the computations. The aim of this paper is to prove the convergence rates (measure of the accuracy) of the trinomial tree methods for the option pricing under regime-switching models.

1. Introduction

Markov regime switching models were first introduced by Hamilton [1] and recently have become popular in financial applications including equity options [2–15], bond prices and interest rate derivatives [16–18], portfolio selection [19], trading rules [20–24], and others. The Markov regime switching models allow the model parameters (drift and volatility coefficients) to depend on a Markov chain which can reflect the information of the market environments and at the same time preserve the simplicity of the models. However when the model parameters are governed by the Markov chain, the valuation of the options becomes complex.

Trinomial tree methods are constructed by Boyle [25] for option pricing with single underlying asset using moments matching techniques. Later the approach is extended to the option pricing with two underlying assets. Tian [26] presents equal probability (1/3) trees with two different parameterizations for recombining trinomial tree and also another parameterization based on the idea of matching the first four moments. Rubinstein [27] explores that the trinomial tree can be constructed by viewing two steps of a binomial tree in combination as a single step of a trinomial tree.

Recently the trinomial tree methods are developed to the option pricing with regime-switching. Liu [11,18] develops a linear tree for a regime-switching geometric Brownian motion model and extends it to a class of regime-switching mean-reverting models that have been frequently used for stochastic interest rates, energy and commodity prices. Liu and Zhao [12] develop a tree method for option pricing with two underlying assets under regime-switching models. Yuen and Yang [14,15] construct an efficient trinomial tree method for option pricing in Markov regime-switching models and use the method to price Asian options and equity-indexed annuities. The mentioned tree methods are shown to be very efficient via the numerical examples. However there are no theoretical results of the convergence rates for the proposed tree methods in the literature. In fact, the theoretical results of the convergence rates are very important and crucial to guarantee the reliability and correctness in use of the tree methods.

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In this paper we prove the convergence rates of trinomial tree methods for pricing options with regime switching. We only focus on the efficient trinomial tree methods proposed by Yuen and Yang [14], instead of all the variants mentioned above, since the results for the variants can be obtained with a similar idea.

2. Trinomial tree methods for regime switching models

In the following, we describe the regime switching models and the trinomial methods of Yuen and Yang [14] for pricing the European options. For ease of exposition, only the two-states regime switching models are studied in this paper and the many-states regime switching models can be analyzed similarly without essential difficulties.

Let the underlying asset prices S_t follow a two-states regime switching model under risk-neutral measure:

$$\frac{dS_t}{S_t} = r\,(X(t))\,dt + \sigma\,(X(t))\,dW_t, \tag{1}$$

where W_t is a standard Brownian motion, $X(t)$ is a continuous-time Markov chain with two states (x_1, x_2). Assume that at each state $X(t) = x_i,\ i = 1, 2$, the interest rates $r\,(x_i) = r_i \geq 0$ and volatility $\sigma\,(x_i) = \sigma_i$ for $i = 1, 2$ are constants. Let $\mathrm{A} = \left(a_{ij}\right)_{i,j=1,2}$ be the generator matrix of the Markov chain process whose elements are constants satisfying $a_{ij} \geq 0$ for $i \neq j$ and $a_{i1} + a_{i2} = 0$ for $i = 1, 2$. Then from [13], the value of European option $V(S, t, i)$ with maturity date T and payoff $f(S_T)$ satisfies the following PDEs

$$\begin{aligned}&\frac{\partial V(S,t,i)}{\partial t} + \frac{1}{2}\sigma_i^2 S^2 \frac{\partial^2 V(S,t,i)}{\partial S^2} + r_i S \frac{\partial V(S,t,i)}{\partial S}\\ &\quad - r_i V(S,t,i) + a_{i1} V(S,t,1) + a_{i2} V(S,t,2) = 0, \quad i = 1, 2,\end{aligned} \tag{2}$$

with terminal condition $V(S, T, i) = f(S),\ i = 1, 2$. Let $\Delta t = T/n$ be the time step-size. Then for all the regimes, the jump rations of the lattice are taken as

$$u = e^{\sigma\sqrt{\Delta t}}, \qquad m = 1, \qquad d = e^{-\sigma\sqrt{\Delta t}}, \tag{3}$$

where σ must satisfy

$$\sigma > \max_{i=1,2}\{\sigma_i\}$$

such that the risk-neutral probability measure exists. As suggested by Yuen and Yang [14], one possible value is

$$\sigma = \max_{i=1,2}\{\sigma_i\} + (\sqrt{1.5} - 1)\bar{\sigma},$$

where $\bar{\sigma}$ is the arithmetic mean or the root mean square of $\sigma_i,\ i = 1, 2$. For regime i, let $\pi_u^i,\ \pi_m^i,\ \pi_d^i$ be risk neutral probabilities corresponding to when the stock price increases, remains the same and decreases, respectively. Then the values of the probabilities are given by, for $i = 1, 2$,

$$\pi_m^i = 1 - \frac{1}{\lambda_i^2}, \tag{4}$$

$$\pi_u^i = \frac{e^{r_i\Delta t} - e^{-\sigma\sqrt{\Delta t}} - \left(1 - 1/\lambda_i^2\right)\left(1 - e^{-\sigma\sqrt{\Delta t}}\right)}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}, \tag{5}$$

$$\pi_d^i = \frac{e^{\sigma\sqrt{\Delta t}} - e^{r_i\Delta t} - \left(1 - 1/\lambda_i^2\right)\left(e^{\sigma\sqrt{\Delta t}} - 1\right)}{e^{\sigma\sqrt{\Delta t}} - e^{-\sigma\sqrt{\Delta t}}}, \tag{6}$$

where $\lambda_i = \sigma/\sigma_i$.

Let $S_{j+1} = uS_j$ and $S_{j-1} = dS_j$ and denote $V^k(S_j, i)$ by the trinomial approximation of the European options for regime i at asset price S_j and time $t_k = k\Delta t$. Then for the trinomial trees (3)–(6), the trinomial value of European options for regime i can be recursively calculated by

$$V^k(S_j, i) = e^{-r_i\Delta t} \sum_{\ell=1}^{2} p_{i\ell}\left(\pi_u^i V^{k+1}(S_{j+1}, \ell) + \pi_m^i V^{k+1}(S_j, \ell) + \pi_d^i V^{k+1}(S_{j-1}, \ell)\right), \tag{7}$$

with $V^n(S_j, i) = f(S_j)$ for $j = -(n-1), \ldots, -1, 0, 1, \ldots, n-1$, in which $p_{i\ell}$ is the transition probability from regime state i to state ℓ for the time interval with length Δt which is given by

$$\begin{pmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{pmatrix} = e^{\mathrm{A}\Delta t} = \mathrm{I} + \sum_{l=1}^{\infty} (\Delta t)^l \mathrm{A}^l / l!, \tag{8}$$

where I is the identity matrix and A is the generator matrix of the Markov chain process.