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Sensitivity of non-autonomous discrete dynamical systems

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ABSTRACT

In this paper, the sensitivity for non-autonomous discrete systems is investigated. First of all, two sufficient conditions of sensitivity for general non-autonomous dynamical systems are presented. At the same time, one stronger form of sensitivity, that is, cofinite sensitivity, is introduced for non-autonomous systems. Two sufficient conditions of cofinite sensitivity for general non-autonomous dynamical systems are presented. We generalized the result of sensitivity and strong sensitivity for autonomous discrete systems to general non-autonomous discrete systems, and the conditions in this paper are weaker than the correlated conditions of autonomous discrete systems.

1. Introduction

Chaos means a random-like behavior (intrinsic randomness) in deterministic systems without any stochastic factors. It is a universal dynamical behavior of nonlinear dynamical systems and one of the central topics of research on nonlinear science. Meanwhile, chaos has a global and essential effect on the development of nonlinear dynamics. Sensitivity is a key ingredient of chaos. Therefore, the research on sensitivity has attracted a lot of scholars' attention [1–8]. In 1992, Banks et al. proved that the two former conditions (transitivity and dense periodic points set) of Devaney chaos imply the last condition—sensitivity [2]. In 1993, Glasner and Weiss got a stronger result: any transitive and non-minimal dynamical system whose almost periodic points are dense in the phase space is sensitive [3]. In 2002, Abraham et al. studied the sensitivity from the viewpoint of ergodicity [1]. They proved that if a measure-preserving map T with full measure support on a metric probability space is either strong-mixing or topologically mixing, weak-mixing with some other conditions, then it is sensitive. In 2010, Li and the second author of the present paper introduced a definition of topologically strongly ergodic and proved that if a measure-preserving map T with full measure support on a metric probability space is topologically strongly ergodic, then T is sensitive [6]. Their result weakened Abraham's conditions in [1]. For other results about sensitivity, we refer to [5,8,9].

The "largeness" of the time set where sensitivity happens can be regarded as a measure of how sensitive the system is. On account of this reason, in 2007, Moothathu proposed three stronger forms of sensitivity: syndetic sensitivity, cofinite sensitivity, and multi-sensitivity [10]. They gave some sufficient conditions of stronger sensitivity and a counter example which is not syndetically sensitive. Recently, Li with the second author of the present paper proposed another stronger form of sensitivity—ergodic sensitivity and gave some sufficient conditions of four stronger sensitivity for measure-preserving map and semi-flow on probability spaces [7].

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In the present paper, we investigate the sensitivity for non-autonomous discrete systems. We introduce one stronger form of sensitivity for non-autonomous systems, that is, cofinite sensitivity. At the same time, we give two sufficient conditions of sensitivity and two sufficient conditions of cofinite sensitivity for general non-autonomous dynamical systems. For other results about chaos for non-autonomous discrete dynamical systems, we refer to [13,14] and references cited therein.

The rest of the paper is organized as follows. In Section 2, some basic concepts are given. In Section 3, sensitivity and cofinite sensitivity for non-autonomous discrete systems are discussed.

2. Preliminaries

In this section, we give some basic concepts.

We first introduce some notations. Now, we consider the following non-autonomous discrete dynamical system:

$$x_{n+1} = T_n(x_n), \quad n \geq 0, \tag{2.1}$$

where D_n is a subset of metric space (X, d) and $T_n : D_n \to D_{n+1}$ is a map. For convenience, denote $\mathcal{T} = \{T_n\}_{n=0}^{\infty}$.

Definition 2.1 (*[11, Definition 2.3]*)**.** Let A be a nonempty subset of D_0. System (2.1) is said to have sensitive dependence on initial conditions in A if there exists a constant $\delta_0 > 0$ such that for any $x_0 \in A$ and any neighborhood U of x_0, there exists $y_0 \in A \cap U$ and a positive integer n such that $d(x_n, y_n) > \delta_0$, where $\{x_i\}_{i=0}^{\infty}$ and $\{y_i\}_{i=0}^{\infty}$ are the orbits of system (2.1) starting from x_0 and y_0, respectively. The constant $\delta_0 > 0$ is called a sensitivity constant of system (2.1) in A.

Let $V \subset D_0$ be a nonempty set, $\mathbf{N}$ the positive integer set, and $\delta > 0$. Denote

$$N_{\mathcal{T}}(V, \delta) := \{n \in \mathbf{N} : \text{ there exist } x, y \in V \text{ such that } d(T_0^n(x), T_0^n(y)) > \delta\}.$$

Then, the sensitive dependence on initial conditions for system (2.1) can be described by $N_{\mathcal{T}}(V, \delta)$. That is, system (2.1) is called sensitive dependence in D_0 if there is a constant $\delta > 0$ such that for any nonempty relative open set V of D_0, $N_{\mathcal{T}}(V, \delta)$ is nonempty.

Definition 2.2. Let V be a nonempty subset of D_0. System (2.1) is said to be topologically mixing in V if, for any two nonempty relative open subsets U_0 and V_0 with respect to V, there exists a positive integer N such that for any $n \geq N$, $U_n \bigcap V_0 \neq \emptyset$, where $U_{i+1} = T_i(U_i)$, $0 \leq i \leq n-1$.

For any two nonempty relative open sets U, V of D_0, we denote

$$N_{\mathcal{T}}(U, V) = \{n \in \mathbf{N} : T_0^n(U) \cap V \neq \emptyset\},$$

where $T_0^n(U) = T_{n-1} \circ \cdots \circ T_0(U)$. Then, the topologically mixing for system (2.1) can be described by $N_{\mathcal{T}}(U_0, V_0)$ as follows: system (2.1) is called topologically mixing in $V \in D_0$ if for any two nonempty relative open sets U_0 and V_0 of V, there is a positive integer N such that $N_{\mathcal{T}}(U_0, V_0) \supset [N, +\infty) \cap \mathbf{N}$.

Let $|N_{\mathcal{T}}(U, V)|$ be the cardinal number of the set $N_{\mathcal{T}}(U, V)$. We call

$$\limsup_{n\to\infty} \frac{|N_{\mathcal{T}}(U, V) \cap N_n|}{n}$$

the upper density of $N_{\mathcal{T}}(U, V)$, where $N_n = \{0, 1, \ldots, n-1\}$.

Inspired by the ideas given in [7,10], we introduce the following concept of cofinitely sensitivity for non-autonomous discrete systems.

Definition 2.3. System (2.1) is called cofinitely sensitive if there exists a constant $\delta > 0$ such that for any nonempty relative open set V of D_0, there exists an $N \geq 1$ such that $N_{\mathcal{T}}(V, \delta) \supset [N, +\infty) \cap \mathbf{N}$, while δ is called a sensitive constant.

Obviously, the cofinitely sensitivity implies sensitivity.

Definition 2.4. System (2.1) or a family of maps $\mathcal{T}$ is called topologically strong ergodic in D_0 if for any two nonempty relative open sets U, V of D_0, the upper density of $N_{\mathcal{T}}(U, V)$ equals 1.

Let $\mathscr{B}(X)$ be the smallest σ-algebra and satisfy $D_n \in \mathscr{B}(X)$, $n \geq 0$. Assume that μ is a measure of measurable space $(X, \mathscr{B}(X))$.

Definition 2.5. We say that system (2.1) is measure-preserving or $\mathcal{T}$ is a family of measure-preserving maps if T_n is measure-preserving on the measurable space $(X, \mathscr{B}(X), \mu)$ for each $n \geq 0$.

Definition 2.6. System (2.1) or a family of measure-preserving maps $\mathcal{T}$ is called strongly mixing in D_0 if for any $A, B \in \mathscr{B}(X)$,

$$\lim_{n\to\infty} \mu((A \cap D_0) \cap T_0^{-n}(B \cap D_0)) = \mu(A \cap D_0)\mu(B \cap D_0)$$

holds, where $T_0^{-n} = (T_0^n)^{-1}$.

Remark 2.1. Definitions 2.5 and 2.6 generalized the concepts given in [12].