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Time of death estimation from temperature readings only: A Laplace transform approach

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ABSTRACT

Time of death estimation is a fundamental problem in forensic medicine. A popular estimation method is by body cooling. In this article, we propose a compartment model for body cooling and that includes the well-known Marshall–Hoare model as a particular case. Using the Laplace transform, we give a new method for estimating the time of death from temperature readings only. We present the results of numerical simulations applied to theoretical and experimental data to show the accuracy of the proposed method.

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1. Introduction

Estimation of the time of death (TOD) is one of the most important problems in the forensic sciences. Such estimations are especially important in the investigation of criminal or suspicious deaths. Mathur and Agrawal [1] gave a recent overview of different methods used in TOD estimation. Body temperature is often used in estimating the TOD; see Knight [2] for an earlier chronological survey of the development of concepts and techniques in using body temperature for TOD estimation. As skin temperature readings are unreliable, measurements from the rectum or liver are usually taken, although Kaliszan [3] gave practical applications of eye temperature measurements for TOD estimation in casework.

Due to the large thermal mass of the corpse, rectal temperature decay is slower during the first few hours after death, thus giving rise to a so-called postmortem plateau and a sigmoidal temperature distribution. This phenomenon has been observed since the 19th century [4].

Rainy [4] made the first attempt to introduce mathematical concepts to study postmortem body cooling. He used sequential rectal temperature measurements of corpses to calculate the rate of cooling per hour. Rainy also considered Newton's law of cooling (namely, the rate of loss of heat from the surface of a solid body to the surrounding fluid is directly proportional to the difference in temperatures between the surface and the fluid) and stated that it is not absolutely correct when applied to human bodies.

However, as pointed out by Brown and Marshall [5], Newton's law of cooling is valid for the human corpse; it is in deriving a useful equation from it that most authors have erred by assuming a set of conditions whereby a body is postulated to be "thermally thin". This assumption is not valid as can be verified from experimental results.

The ordinary differential equation (ODE) that is usually derived from Newton's law of cooling yields a single exponential model, which cannot capture the initial temperature lag. Marshall and Hoare [6] (see also [7,8]) proposed a double

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exponential model which was found to reproduce the sigmoidal cooling curves obtained from experiments. Later, Brown and Marshall [5] gave further mathematical justifications for the lag in early cooling.

The rate constants in the Marshall–Hoare model depend on factors such as body mass, height, clothing, presence of wind, wetness, etc. By averaging out some of these factors and using sound statistical analysis, Henßge [9] estimated these rate constants as functions of the body mass. The usefulness of Henßge's approximations is that it allows one to estimate the TOD with one temperature reading and an accurate estimate of the weight [10]. Henßge and coauthors also developed a nomogram to aid in field calculations [9,11,12].

Since thermocouple detectors and computerized recording have become commonplace [2], there has been a trend to not explicitly model the effects of external factors but instead consider self-contained calculations derived from several temperature readings [13,14]. The same idea is also found in the papers by Green and Wright [15,16], where they started with a single exponential model and replaced the constant cooling rate by a time-varying function. Nokes et al. [17] presented eight commonly used temperature-based techniques for TOD estimation. We remark that TOD estimation can be viewed as an inverse problem: given a set of temperature readings, estimate the parameters that will fit the model to the data.

Compartment models are used in many fields, e.g., pharmacokinetics, epidemiology, systems theory, engineering, and the social sciences [18]. A compartment model is a way of describing how materials or energies are transmitted among the different compartments. In this article, we propose a single compartment model for postmortem body cooling, treating the rectum as the compartment. Our model includes an arbitrary, time-dependent function that is related to the heat energy entering the compartment due to the body's thermal conductivity. We identify a set of sufficient conditions that this function must satisfy in order to model the postmortem plateau while also incorporating the effect of Newton's law of cooling and a sigmoidal profile for the cooling curve. The Marshall–Hoare model [6] is recovered for a particularly simple choice of this arbitrary function. Following an approach based on the Laplace transform, we then propose a method to estimate the TOD that takes as input a set of temperature readings only. We illustrate the procedure using theoretical data, as well as experimental data obtained in [19].

2. A compartment model for postmortem body cooling

Brown and Marshall [5] discussed two propositions to account for the sigmoidal profile of the temperature distribution. The first proposition suggests that the slow early postmortem cooling rate is due to the continuing production of heat by internal heat sources of the body. However, Lundquist's results [20] seem to indicate that heat produced after death contributes to, but not totally accounts for, the sigmoidal shape.

The second proposition is that the slow initial cooling is more likely due to the thermal conductivity and thermal capacity of the human body. Most of the heat removed from the dead body is due to convection from the skin according to Newton's law of cooling. As surface heat is lost, heat is transferred to the surface from the central body core by conduction, thus creating a temperature difference between the surface and subjacent layers of the body. When a layer of the body starts to cool by conduction of some of its heat to the body surface, it creates a condition whereby heat flows into it from the hotter region immediately deep to it. Hence Marshall and Brown's conclusion was that heat flowing out to the surface from any layer of the body consists partly of heat lost by the layer itself and partly of heat flowing through the layer from the more central regions of the body [5].

Keeping the above considerations in mind, we construct a compartment model for postmortem body cooling, where the compartment is the rectum. A basic balance law states that the rate of change of heat energy in a compartment is equal to the heat energy going into the compartment per unit time minus the heat energy going out of the compartment per unit time. Let c be the specific heat and let m be the mass. If Q is the heat energy in the compartment and T is the temperature, then $Q = cmT$ and

$$\frac{\mathrm{d}Q}{\mathrm{d}t} = cm\frac{\mathrm{d}T}{\mathrm{d}t} = Q_{\mathrm{in}} - Q_{\mathrm{out}},$$

where Q_{in} is the heat energy going into the compartment per unit time and Q_{out} is the heat energy going out of the compartment per unit time. Using Newton's law of cooling, Q_{out} is proportional to the difference between the temperature at the rectal surface and the ambient temperature, i.e.,

$$Q_{\mathrm{out}}(t) = hA[T(t) - T_\infty],$$

where h is the heat transfer coefficient, A is the surface area, and T_∞ is the constant ambient temperature. Therefore

$$\frac{\mathrm{d}T}{\mathrm{d}t} = q(t) - Z(T - T_\infty), \tag{2.1}$$

where

$$q(t) = \frac{1}{cm}Q_{\mathrm{in}}(t), \quad Z = \frac{hA}{cm}.$$

Thus (2.1) is a general single compartment model for postmortem body cooling.

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