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Coupled flow and heat transfer in viscoelastic fluid with Cattaneo–Christov heat flux model

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ABSTRACT

This letter presents a research for coupled flow and heat transfer of an upper-convected Maxwell fluid above a stretching plate with velocity slip boundary. Unlike most classical works, the new heat flux model, which is recently proposed by Christov, is employed. Analytical solutions are obtained by using the homotopy analysis method (HAM). The effects of elasticity number, slip coefficient, the relaxation time of the heat flux and the Prandtl number on velocity and temperature fields are analyzed. A comparison of Fourier's Law and the Cattaneo–Christov heat flux model is also presented.

1. Introduction

Heat transfer is a widespread natural phenomenon of nature, as long as there is a temperature difference between objects or between different parts of the same object, there will be heat transfer phenomenon. Therefore, a considerable attention has been devoted to predict the heat transport behavior. In 1822, Fourier [1] firstly proposed the famous law of heat conduction law. Though this law is an empirical formula, it provides a way to study heat transfer phenomenon and becomes the basis to study heat conduction in the subsequent two centuries. Cattaneo [2] proposed a modified Fourier heat conduction law, by adding a relaxation time term, to overcome the 'paradox of heat conduction', which is 'any initial disturbance is felt instantly throughout the whole of the medium under consideration' [3,4]. Christov [3] proposed the frame-indifferent generalization of Cattaneo' law by using the Oldroyds' upper-convected derivative [5], who derived a single temperature governing equation. The uniqueness and structural stability of the solutions for the temperature governing equations with Cattaneo–Christov heat flux model in some initial and boundary problem, have been proved by Tibullo and Zampoli [4], Ciarletta and Straughan [6]. Straughan [7] and Haddad [8] obtained the numerical solutions for thermal convection of incompressible Newtonian fluid with Cattaneo–Christov heat flux model by using the D^2 Chebyshev tau method.

The Maxwell fluid is an important class of viscoelastic fluid models. In the past few decades, a large number of research achievements regarding Maxwell fluid model have been published. Among these literatures, boundary layer problems of upper-convected Maxwell fluid have obtained attention. Choi et al. [9] studied the steady suction flow, the effects of viscoelasticity and inertia in a porous channel were considered. Sadeghy et al. [10] investigated Sakiadis flow by employing three different mathematical methods and pointed out the wall skin friction coefficient decreases when increasing the value of Deborah number. MHD boundary layer flows in porous objects or space were discussed in [11–14] and stagnation-point

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flows of Maxwell fluid were studied in [15–18]. Hayat et al. [19] investigated chemical reaction effects on mass transfer and MHD boundary layer flow above a porous shrinking sheet. The convective boundary conditions and viscous dissipation heat transfer over a moving surface were considered in [20]. Aliakbar et al. [21] investigated heat transfer of an upper-convected Maxwell fluid with viscous dissipation and thermal radiation.

The homotopy analysis method proposed by Liao [22,23], is an effective mathematical method to solve nonlinear differential equations. Analytical solution has important significance in the study of physical problems. According to current knowledge of the authors, no literature presented the analytical solutions for the temperature governing equation with Cattaneo–Christov heat flux model. Therefore, we will firstly obtain the analytical solutions by using homotopy analysis method.

The main purpose of this letter is to study the coupled flow and heat transfer of boundary layer in viscoelastic fluid with the upper-convected Maxwell model and Cattaneo–Christov heat flux model, the similarity transformation and homotopy analysis method are used with a view to obtain analytical solutions. The effects of involved parameters on the velocity and temperature fields are analyzed and discussed.

2. The basic equations

Consider a two dimensional steady boundary layer flow of the Maxwell fluid over a plate, in the absence of the pressure gradient, the governing equation for expressing conservation of mass, momentum can be written as [10,20]

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}+\lambda_1\left(u^2\frac{\partial^2 u}{\partial x^2}+v^2\frac{\partial^2 u}{\partial y^2}+2uv\frac{\partial^2 u}{\partial x\partial y}\right)=\nu\frac{\partial^2 u}{\partial y^2}. \tag{2}$$

The boundary conditions with velocity slip on boundary are:

$$u=ax+\lambda_0\frac{2-\sigma_v}{\sigma_v}\frac{\partial u}{\partial y}\Big|_{y=0}, \quad v=0 \text{ at } y=0,\ u\to 0 \text{ as } y\to\infty \tag{3}$$

where a is positive constant, σ_v is the tangential momentum accommodation coefficient, λ_0 is the molecular mean free path.

We use the Cattaneo–Christov heat flux model to study heat transfer of the Maxwell fluid, the heat flux model is the following form [3]

$$\mathbf{q}+\lambda_2\left[\frac{\partial \mathbf{q}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{q}-\mathbf{q}\cdot\nabla\mathbf{V}+(\nabla\cdot\mathbf{V})\,\mathbf{q}\right]=-k\nabla T \tag{4}$$

where $\mathbf{q}$ is heat flux, λ_2 is relaxation time of the heat flux, T is temperature of the Maxwell fluid, k is the thermal conductivity. $\mathbf{V}=(u,v)$ is velocity vector of the Maxwell fluid. When $\lambda_2=0$, Eq. (4) is simplified to Fourier's law. Since the fluid is incompressible, which satisfies $\nabla\cdot\mathbf{V}=0$, Eq. (4) becomes the following form [4]

$$\mathbf{q}+\lambda_2\left(\frac{\partial \mathbf{q}}{\partial t}+\mathbf{V}\cdot\nabla\mathbf{q}-\mathbf{q}\cdot\nabla\mathbf{V}\right)=-k\nabla T. \tag{5}$$

The energy balance equation for the steady boundary layer flow is

$$\rho c_p\mathbf{V}\cdot\nabla T=-\nabla\cdot\mathbf{q}. \tag{6}$$

Eliminating $\mathbf{q}$ between Eqs. (5) and (6), we can obtain the temperature governing equation for the steady flow

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}+\lambda_2\left(u\frac{\partial u}{\partial x}\frac{\partial T}{\partial x}+v\frac{\partial v}{\partial y}\frac{\partial T}{\partial y}+u\frac{\partial v}{\partial x}\frac{\partial T}{\partial y}+v\frac{\partial u}{\partial y}\frac{\partial T}{\partial x}+2uv\frac{\partial^2 T}{\partial x\partial y}+u^2\frac{\partial^2 T}{\partial x^2}+v^2\frac{\partial^2 T}{\partial y^2}\right)=\alpha\frac{\partial^2 T}{\partial y^2} \tag{7}$$

where $\alpha=k/\rho c_p$ is thermal diffusivity. The corresponding boundary conditions are

$$T=T_w \quad \text{at } y=0, \qquad T\to T_\infty \quad \text{as } y\to\infty. \tag{8}$$

Employing the following transformations

$$\eta=y\sqrt{\frac{a}{\nu}}, \qquad \psi=x\sqrt{\nu a}f(\eta), \qquad \theta=\frac{T-T_\infty}{T_w-T_\infty} \tag{9}$$

where ψ is the stream function and has the relationship: $u=\partial\psi/\partial y$, $v=-\partial\psi/\partial x$. We obtain the following nonlinear coupled equations

$$f'''-f'^2+ff''+\beta\left(2ff'f''-f^2f'''\right)=0 \tag{10}$$

$$\frac{1}{\Pr}\theta''+f\theta'-\gamma\left(ff'\theta'+f^2\theta''\right)=0 \tag{11}$$

where $\beta=\lambda_1 a$, $\gamma=\lambda_2 a$, $\Pr=\nu/\alpha$ is the Prandtl number.