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Kinks and travelling wave solutions for Burgers-like equations

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ABSTRACT

In this work we develop a variety of Burgers-like equations. We show that these derived equations share some of the travelling wave solutions of the Burgers equation, and differ in some other solutions.

1. Introduction

The standard Burgers equation reads

$$u_t + uu_x + u_{xx} = 0. \tag{1}$$

This equation is the lowest order approximation for the one-dimensional propagation of weak shock waves in a fluid [1,2]. The Burgers equation models the coupling between dissipation effect u_{xx} and the convection effect uu_x. It is completely integrable and admits multiple-kink solutions. The kink solution of the Burgers equation is given by

$$u(x, t) = 2k\left(1 + \tanh(kx - 2k^2\, t)\right). \tag{2}$$

Moreover, the singular solution

$$u(x, t) = 2k\left(1 + \coth(kx - 2k^2\, t)\right), \tag{3}$$

satisfies the Burgers equation (1).

Studies on nonlinear evolution equations are growing rapidly because these equations describe real features in scientific applications. Powerful methods have been used to determine the solutions with distinct physical structures [1–10]. Examples of the methods that have been used are the Hirota bilinear method, the Bäcklund transformation method, and the inverse scattering method. Moreover, the derivation of new nonlinear equations has attracted much attention recently [11–17]. The new derived equations may describe significant features that are related to the well known models.

It is the aim of this work to formulate new Burgers-like equations, where some of these equation share the same solutions with Burgers equation.

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2. Formulation of the Burgers-like equations

Following [3], a generalized form of an advection–dissipation equation is considered as

$$u_t + Vu_x = \delta u_{xx}, \tag{4}$$

where δ is an arbitrary dimensionless parameter and $V(u, u_x, u_{xx}, \ldots)$ is an arbitrary function. We assume the travelling wave to be of the form

$$u(x,t) = f(x - ct) = f(\xi), \tag{5}$$

that solves the Burgers equation (1), and also solves the advection–dissipation equation (4) for the same speed c. Using $\xi = x - ct$ transforms (1) and (4) to

$$-cf' + ff' + f'' = 0, \tag{6}$$

and

$$-cf' + Vf' - \delta f'' = 0. \tag{7}$$

Eliminating f'' from these two equations, and by noting that $f' \neq 0$, we obtain

$$V = (\delta + 1)c - \delta f. \tag{8}$$

The advection–dissipation equation, or the Burgers-like equation can be simply obtained by using a variety of values of the speed c. From (6), we find

$$c = f + \frac{f''}{f'}. \tag{9}$$

Integrating (6) one time and solving for c we find

$$c = \frac{1}{2}f + \frac{f'}{f}. \tag{10}$$

To determine more values for the speed c, we can differentiate (6) as many times as we want [3]. For example, differentiating (6) once, two times, and three times and solving for c we find

$$c = f + \frac{(f')^2 + f'''}{f''}, \tag{11}$$

$$c = f + \frac{3f'f'' + f^{(iv)}}{f'''}, \tag{12}$$

and

$$c = f + \frac{4f'f''' + 3(f'')^2 + f^{(v)}}{f^{(iv)}}, \tag{13}$$

respectively. By differentiating as many times as we want we can determine many values for c.

Substituting (9)–(13) into (8) gives

$$V_1 = (\delta + 1)\left(f + \frac{f''}{f'}\right) - \delta f, \tag{14}$$

$$V_2 = (\delta + 1)\left(\frac{1}{2}f + \frac{f'}{f}\right) - \delta f, \tag{15}$$

$$V_3 = (\delta + 1)\left(f + \frac{(f')^2 + f'''}{f''}\right) - \delta f, \tag{16}$$

$$V_4 = (\delta + 1)\left(f + \frac{3f'f'' + f^{(iv)}}{f'''}\right) - \delta f, \tag{17}$$

and

$$V_5 = (\delta + 1)\left(f + \frac{4f'f''' + 3(f'')^2 + f^{(v)}}{f^{(iv)}}\right) - \delta f. \tag{18}$$