Applied Mathematics Letters 38 (2014) 180–185

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

An extension result of the opposing mixed convection problem arising in boundary layer theory[☆]

G.C. Yang [*]

College of Applied Mathematics, Chengdu University of Information Technology, Chengdu 610225, Sichuan, PR China

ARTICLE INFO

Article history:
Received 25 March 2014
Received in revised form 1 May 2014
Accepted 1 May 2014
Available online 21 May 2014

Keywords:
Opposing mixed convection
Descending external temperature
Existence of solutions

ABSTRACT

We prove analytically the existence of solutions for the opposing mixed convection problem when the external temperature is descending. The obtained result extends some recent study.

1. Introduction

The following third order boundary value problem

$$f'''(\eta) + (1+\lambda)f(\eta)f''(\eta) + 2\lambda(1-f'(\eta))f'(\eta) = 0 \quad \text{on } \mathbb{R}^+ = [0,\infty), \tag{1.1}$$

$$f(0) = 0, \quad f'(0) = 1+\varepsilon \text{ and } f'(\infty) = 1 \tag{1.2}$$

arises in the study of planar mixed convection boundary layer flows [1,2], where λ is the external temperature parameter of flows, ε is the mixed convection parameter, that is, $\varepsilon = R_a/P_e$ where R_a is the *Rayleigh* number and P_e is the *Péclet* number. The problem (1.1)–(1.2) with $\lambda > 0$ ($\lambda = 0$, $\lambda < 0$) corresponds to the ascending (a constant, the descending) of external temperature of flows respectively; the problem (1.1)–(1.2) with $\varepsilon > 0$ ($\varepsilon = 0$, $\varepsilon < 0$) corresponds to the aiding (the forced, the opposing) mixed convection respectively.

Very recently, authors [3] proved analytically the existence and nonexistence of convex solutions of (1.1)–(1.2) when $\lambda > 0$ and $\varepsilon < -1$ via studying the existence of positive solutions for the integral equation (see (2.1)). The other study only treated the cases of $\lambda = 0$ or $\varepsilon \geq -1$. For example, Guedda [4] for $-1 < \lambda < 0$ and $-1 < \varepsilon < \frac{1}{2}$; Brighi and Hoernel [5] for $\lambda > 0$, $-1 < \varepsilon < 0$ and $\varepsilon > 0$. The problem (1.1)–(1.2) with $\lambda = 0$ is the well-known Blasius equation, one may refer to [6,7] and the references therein.

In this paper, we extend the existence of convex solutions of (1.1)–(1.2) [3–5] to $\lambda < 0$ and $\varepsilon < -1$, that is, we shall prove the following result.

Theorem 1.1. *For $(\lambda,\varepsilon) \in \Lambda^*$, the problem (1.1)–(1.2) has a solution $f \in C^3(\mathbb{R}^+)$, where*

$$\Lambda^* = \left\{(\lambda,\varepsilon) \in \left(-\frac{1}{3}, 0\right) \times (-\infty,-1) : \lambda \text{ and } \varepsilon \text{ satisfy } \frac{3(1+3\lambda)^2}{(1+\lambda)^2} - 128\beta^2 + \frac{256}{3}\beta^3 \geq 0\right\}.$$

☆ Project supported by the National Natural Science Foundation of China (Grant No. 11171046).

* Tel.: +86 2885966353.
E-mail addresses: cuityang@163.com, gcyang@cuit.edu.cn.

http://dx.doi.org/10.1016/j.aml.2014.05.007
0893-9659/

2. Proof of Theorem 1.1

Let

$$\Lambda = \left\{(\lambda, \beta) \in \left(-\frac{1}{3}, 0\right) \times (-\infty, 0) : \lambda \text{ and } \beta \text{ satisfy } \kappa(\lambda, \beta) \geq 0\right\},$$

where $\kappa(\lambda, \beta) = 3\varphi^2(\lambda) - 128\beta^2 + \frac{256}{3}\beta^3$ and $\varphi(\lambda) = \frac{1+3\lambda}{1+\lambda}$ (see Fig. 1).

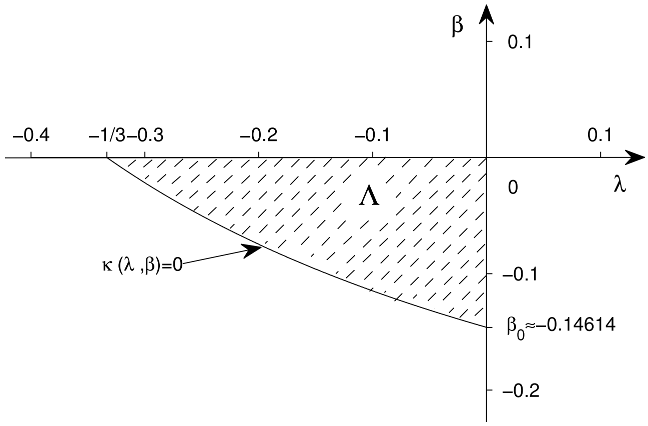

Fig. 1. Figure of Λ—the shaded region.

Remark 2.1. Let $\beta = 1 + \varepsilon$, it is clear that $(\lambda, \varepsilon) \in \Lambda^*$ if and only if $(\lambda, \beta) \in \Lambda$.

Like Theorem 2.1 [3], we prove the following.

Theorem 2.1. *For $(\lambda, \beta) \in \Lambda$, the following integral equation*

$$z(t) = \varphi(\lambda)Az(t) + (1-t)Bz(t), \quad \beta \leq t \leq 1 \tag{2.1}$$

has a solution $z \in C[\beta, 1]$ with $z(t) > 0$ for $t \in [\beta, 1)$, where A and B are defined by

$$Az(t) = \int_t^1 \frac{s(1-s)}{z(s)}\, ds \quad \text{and} \quad Bz(t) = \int_\beta^t \frac{s}{z(s)}\, ds \quad \text{for } t \in [\beta, 1).$$

Proof. Let

$$\sigma(\lambda, \beta) = \frac{\varphi(\lambda) - 3\beta^2}{6}, \quad (\lambda, \beta) \in \Lambda,$$

$$c_{\lambda,\beta} = \frac{\sqrt{3}\varphi(\lambda) - \sqrt{\kappa(\lambda, \beta)}}{16}, \quad (\lambda, \beta) \in \Lambda.$$

Then the following facts hold:

(P_1) $0 < c_{\lambda,\beta} \leq \frac{\sqrt{3}\varphi(\lambda)}{16} < \frac{\sqrt{3}}{16}$.

(P_2) $\sigma(\lambda, \beta) > c^2_{\lambda,\beta}$.

(P_3) $\frac{\sqrt{3}\varphi(\lambda)}{8} + \frac{-3\beta^2+2\beta^3}{6c_{\lambda,\beta}} = c_{\lambda,\beta}$.

In fact, $(\lambda, \beta) \in \Lambda$ implies $\beta < 0$ and $0 < \varphi(\lambda) < 1$. By $\kappa(\lambda, \beta) < 3\varphi^2(\lambda)$, we see $c_{\lambda,\beta} > 0$ and $c_{\lambda,\beta} \leq \frac{\sqrt{3}\varphi(\lambda)}{16} < \frac{\sqrt{3}}{16}$, *i.e.*, (P_1) holds. $\kappa(\lambda, \beta) \geq 0$ and $\beta < 0$ imply $3\varphi^2(\lambda) \geq 128\beta^2$, and we conclude therefore that

$$\sigma(\lambda, \beta) = \frac{\varphi(\lambda) - 3\beta^2}{6} \geq \frac{\varphi(\lambda) - 3 \times \frac{3}{128}\varphi^2(\lambda)}{6} \geq \frac{\varphi^2(\lambda) - 3 \times \frac{3}{128}\varphi^2(\lambda)}{6} = \frac{119\varphi^2(\lambda)}{768}.$$

It follows from (P_1) that $c^2_{\lambda,\beta} < \frac{3\varphi^2(\lambda)}{256}$ and then (P_2) holds. By direct verification, we know that (P_3) holds.

Notation

$$K = \{z : z \in C[\beta, 1], \|z\| \leq R\},$$

where $R = \frac{(\varphi(\lambda)+1)(6\beta^2-4\beta^3+3)}{12c_{\lambda,\beta}}$, and $C[\beta, 1]$ is the space of continuous functions on $[\beta, 1]$ with the norm $\|z\| = \max\{|z(t)| : t \in C[\beta, 1]\}$.