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Piecewise reproducing kernel method for singularly perturbed delay initial value problems

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ABSTRACT

A direct application of the reproducing kernel method presented in the previous works cannot yield accurate approximate solutions for singularly perturbed delay differential equations. In this letter, we construct a new numerical method called piecewise reproducing kernel method for singularly perturbed delay initial value problems. Numerical results show that the present method does not share the drawback of standard reproducing kernel method and is an effective method for the considered singularly perturbed delay initial value problems.

1. Introduction

Many different phenomena can arise in singularly perturbed delay differential equations. Such problems are particularly interesting and challenging. Most of singularly perturbed delay differential equations do not have exact solutions, so numerical techniques must be used. There has been a growing interest to develop numerical techniques for singularly perturbed delay differential equations.

Amiraliyev, Erdogan [1,2] proposed uniform numerical methods for singularly perturbed initial value problem for a linear first-order delay differential equation. Amiraliyeva, Erdogan and Amiraliyev [3] developed an exponentially fitted difference scheme for solving singularly perturbed initial value problem for a quasi-linear second order delay differential equations. In [4–10], the authors considered some approximating aspects for singularly perturbed delay boundary value problems.

In the previous works [11–16], the authors presented the reproducing kernel method (RKM) for solving differential equations for initial and boundary value problems. However, the direct application of the method cannot produce good numerical results for singularly perturbed delay differential equations. The main objective of this work is to develop an effective numerical method for a class of singularly perturbed delay initial value problems by carrying reproducing kernel method in a piecewise fashion.

Motivated by the work of [1,2], consider the following singularly perturbed delay differential problem:

$$\begin{cases} \varepsilon u'(t) + a(t)u(t) + b(t)u(t-r) = f(t), \quad t \in I, \\ u(t) = \varphi(t), t \in I_0 = (-r, 0], \end{cases} \tag{1.1}$$

where $0 < \varepsilon \ll 1$, r is a constant delay, $I = (0, T] = \bigcup_{p=1}^{m} I_p$, $I_p = (r_{p-1}, r_p]$, $r_p = pr$, $a(t) \geq \alpha > 0$, $b(t)$, $\varphi(t)$ and $f(t)$ are assumed to be sufficiently smooth functions such that the solution of (1.1) exhibits boundary layers on the right side of each point $t = r_p$ $(p = 1, 2, \ldots, m-1)$.

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The rest of the paper is organized as follows. In the next section, the piecewise reproducing kernel method for problem (1.1) is proposed. Error analysis is discussed in Section 3. The numerical tests are provided in Section 4. Section 5 ends this paper with a brief conclusion.

2. Piecewise reproducing kernel method

Firstly, we introduce a reproducing kernel space $W^n[a, b]$, $(n \geq 2)$ which will be used in solving the initial value problems by making use of the RKM.

Definition 2.1. $W^n[a, b] = \{u(x) \mid u^{(n-1)}(x)$ is an absolutely continuous real value function, $u^{(n)}(x) \in L^2[a, b], u(a) = 0\}$. The inner product and norm in $W^n[a, b]$ are given respectively by

$$(u, v)_n = \sum_{i=0}^{n-1} u^{(i)}(a)v^{(i)}(a) + \int_a^b u^{(n)}(x)v^{(n)}(x)dx$$

and

$$\|u\| = \sqrt{(u, u)_n}, \quad u, v \in W^n[a, b].$$

Theorem 2.1. *$W^n[a, b]$ is a reproducing kernel space and its reproducing kernel is*

$$k(x, y) = \begin{cases} k_1(x, y), & y \leq x, \\ k_1(y, x), & y > x, \end{cases} \tag{2.1}$$

where $k_1(x, y) = \begin{cases} -\frac{1}{6}(a-y)\left(2a^2 - a(3x+y+6) + 3x(y+2) - y^2\right), n = 2, \\ -\frac{1}{120}(a-y)(6a^4 - 3a^3(5x+3y+10) + a^2(10x^2 + 5x(5y+12) + y(y+30)) \\ -a(10x^2(2y+3) + 5x(y+12)y - y^3 + 120) + 10x^2y(y+3) - 5x(y^3-24) + y^4), n = 3. \end{cases}$

For the proof, please refer to [11,13].

We will solve initial value problem (1.1) using the RKM in a piecewise fashion.

We first divide $[0, T]$ into M subintervals $[t_j, t_{j+1}]$, $j = 0, 1, \ldots, M-1$, with $t_0 = 0$ and $t_M = T$. Also, it is required that $\{r_j\}_{j=1}^m \subset \{t_i\}_{i=1}^M$. Denote by h_i the length of the ith subinterval, i.e. $h_i = |t_i - t_{i-1}|$. Then we obtain the approximate solutions on every interval $[t_{i-1}, t_i]$ by using the RKM.

On subinterval $[t_0, t_1] \subset I_1$, (1.1) is reduced to

$$\begin{cases} \varepsilon u'(t) + a(t)u(t) = f(t) - b(t)\varphi(t-r), \\ u(t_0) = \varphi(t_0). \end{cases} \tag{2.2}$$

Solving (2.2) by the RKM and taking $N+1$ equidistant nodes on interval $[t_0, t_1]$, we obtain the approximate solution $u_{1,N}(t)$ of (1.1) on $[t_0, t_1]$. Obviously, $u_{1,N}(t)$ can provide an approximate value of $u(t_1)$.

On subinterval $[t_1, t_2] \subset I_1$, the approximate form of (2.2) is

$$\begin{cases} \varepsilon u'(t) + a(t)u(t) = f(t) - b(t)\varphi(t-r), \\ u(t_1) = u_{1,N}(t_1). \end{cases} \tag{2.3}$$

We can also obtain the approximate solution $u_{2,N}(t)$ of (1.1) on the second subinterval $[t_1, t_2]$ by using the RKM.

In the same manner, we can obtain the approximate solutions on all subintervals of I_1. Then the approximate solution of (1.1) on interval I_1 is obtained and denoted by $U_1(t)$.

On subinterval $[t_i, t_{i+1}] \subset I_2$, (1.1) is approximated by

$$\begin{cases} \varepsilon u'(t) + a(t)u(t) = f(t) - b(t)U_1(t-r), \\ u(t_i) = u_{i-1,N}(t_i). \end{cases} \tag{2.4}$$

It is easy to obtain the approximate solution $u_{i,N}(t)$ of (1.1) on the subinterval $[t_i, t_{i+1}]$ by using the RKM. After obtaining the approximate solutions of all subintervals of I_2, the approximate solution of (1.1) on interval I_2 is obtained and denoted by $U_2(t)$.

Similarly, we can get approximate solutions $U_k(t)$ to (1.1) on I_k, , $k = 3, 4, \ldots, m$.

After obtaining the approximate solutions of all subintervals I_p, $p = 1, 2, \ldots, m$, these solutions are combined to obtain the approximate solution $U(t)$ of (1.1) on the entire interval I. Clearly, $U(t)$ is continuous on $I = [0, T]$.

3. Error analysis

Define $Lu(t) = \varepsilon u'(t) + a(t)u(t)$. From [17], we have the following two theorems.