Applied Mathematics Letters 31 (2014) 46–51

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Decay rate estimates for a weak viscoelastic beam equation with time-varying delay[☆]

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ARTICLE INFO

Article history:
Received 23 December 2013
Received in revised form 10 February 2014
Accepted 10 February 2014
Available online 19 February 2014

Keywords:
Beam equation
Time-varying delay
Weak viscoelasticity
General decay rate

ABSTRACT

We consider a weak viscoelastic beam equation with internal time-varying delay

$$u_{tt} + \Delta^2 u - M(\|\nabla u\|^2)\Delta u + \sigma(t)\int_0^t g(t-s)\Delta u(s)ds$$
$$+a_0 u_t + a_1 u_t(x, t-\tau(t)) = 0.$$

By introducing suitable energy and Lyapunov functionals, we establish general decay estimates for the energy, which depends on the behavior of both σ and g.

1. Introduction

This work is concerned with decay properties of solutions for the following weak viscoelastic beam equation with time-varying delay

$$\begin{cases} u_{tt}(x,t) + \Delta^2 u(x,t) - M(\|\nabla u(t)\|^2)\Delta u(x,t) + \sigma(t)\displaystyle\int_0^t g(t-s)\Delta u(x,s)ds \\ \quad + a_0 u_t(x,t) + a_1 u_t(x, t-\tau(t)) = 0, \quad (x,t) \in \Omega \times (0,\infty), \\ u(x,t) = \dfrac{\partial u(x,t)}{\partial \nu} = 0, \quad (x,t) \in \Gamma \times (0,\infty), \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega, \\ u_t(x,t) = f_0(x,t), \quad (x,t) \in \Omega \times [-\tau(0), 0), \end{cases} \tag{1.1}$$

where Ω is a bounded domain in $\mathbb{R}^n (n \geq 2)$ with a sufficiently smooth boundary Γ, ν is the unit outward normal to Γ, a_0 is a positive constant, a_1 is a real number, M, σ, and g are real functions, $\tau(t) > 0$ represents the time-varying delay. When $a_1 = 0$ in the first equation of (1.1), that is, in the absence of delay, problem (1.1) was studied by many authors. This equation is a model for vibrations of extensible beams and was introduced by Woinowsky-Krieger [1] in the form

$$\frac{\partial^2 u}{\partial t^2} + \alpha\frac{\partial^4 u}{\partial x^4} - \left(\beta + \gamma\int_0^L \left(\frac{\partial u}{\partial x}\right)^2 dx\right)\frac{\partial^2 u}{\partial x^2} = 0, \tag{1.2}$$

☆ This research was supported by the Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science and Technology (No. 20110007870).
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http://dx.doi.org/10.1016/j.aml.2014.02.005
0893-9659/

where L is the length of the beam, α, β and γ are positive physical constants. Its modeling aspects were also discussed in [2,3]. Some mathematical problems related to (1.2) have been previously studied by several authors [4,5]. As a general form of (1.2), many authors studied the following equation with appropriate boundary and initial conditions:

$$u_{tt}+\alpha\Delta^2u-M\left(\int_\Omega|\nabla u|^2dx\right)\Delta u+\int_0^t g(t-s)\Delta u(s)ds+h(u_t)=0, \tag{1.3}$$

where M, h and g are functions satisfying some conditions (see e.g. [6–12]). Menezes et al. [7] proved existence and uniform decay results of solutions for nonlinear beam degenerate equation (1.3) with $g=0$ and linear damping $h(u_t)=u_t$. Park et al. [9] showed decay rate estimates for the problem (1.3) with $\alpha=1$, $g=0$, $h=0$ and boundary damping. These works are concerned with exponential decay rates. The aim of this work is to prove a general decay result for weak viscoelastic beam equation (1.1) with time-varying delay. Considering the time-varying delay term $a_1u_t(x,t-\tau(t))$, the problem is different from the existing literature. Time delays arise in many applications depending not only on the present state but also on some past occurrences. And the presence of delay may be a source of instability (see e.g. [13,14]). Thus, recently, the control of partial differential equations with time delay effects has become an active area of research (see [15,16,14, 17–19] and references therein). Kafini et al. [15] and Nicaise and Pignotti [14] examined a Timoshenko-type system of thermoelasticity of type III and a wave equation with constant delay, respectively. Mustafa [16] studied a thermoelastic system of second sound with internal time-varying delay. Nicaise and Pignotti [14] showed decay rates for a wave equation with time-varying delay. These works are also concerned with exponential or polynomial decay rates.

On the other hand, Messaoudi [20] considered the viscoelastic equation without the linear damping and time-varying delay of the form

$$u_{tt}-\Delta u+\sigma(t)\int_0^t g(t-s)\Delta u(s)ds=0,$$

under conditions on σ and g such as

$$\sigma(t)>0,\qquad g'(t)\le-\zeta(t)g(t),\qquad \lim_{t\to\infty}\frac{-\sigma'(t)}{\zeta(t)\sigma(t)}=0,$$

where ζ is a nonincreasing and positive function. He obtained general stability by making use of the perturbed energy method. This approach is based on the construction of a suitable functional $\mathcal{L}$, which is equivalent to the corresponding energy, satisfying

$$\frac{d}{dt}\mathcal{L}(t)\le-c\sigma(t)\zeta(t)\mathcal{L}(t)\quad\text{for some } c>0.$$

For other related works, we refer the readers to [21–25] and references therein. Inspired by these results, we prove a general decay result for problem (1.1) making use of the idea of [20,17]. The plan of this paper is as follows. In Section 2, we give some notations and material needed for our work. In Section 3, we derive a general decay estimate of the energy.

2. Preliminaries

In this section, we present some material needed in the proof of our results. Throughout this paper, we denote $\|\cdot\|_X$ the norm of a Banach space X. We use standard functional spaces $L^2(\Omega)$ endowed with the inner product $(u,v)=\int_\Omega u(x)v(x)dx$ and the induced norm $\|u\|_{L^2(\Omega)}=\sqrt{(u,u)}$. We also consider the Sobolev space $H^2(\Omega)$ endowed with the scalar product $(u,v)_{H^2(\Omega)}=(u,v)+(\Delta u,\Delta v)$. We define the subspace of $H^2(\Omega)$, denoted by $H_0^2(\Omega)$, as the closure of $C_0^\infty(\Omega)$ in the strong topology of $H^2(\Omega)$. This space endowed with the norm induced by the scalar product $(u,v)_{H_0^2(\Omega)}=(\Delta u,\Delta v)$ is a Hilbert pace. For simplicity, we denote $\|\cdot\|_{L^2(\Omega)}$ by $\|\cdot\|$. Let λ_1 be the first eigenvalue of the spectral Dirichlet problem $\Delta^2u+\lambda\Delta u=0$ in Ω, $u=\frac{\partial u}{\partial\nu}=0$ on Γ (see [26]).

Concerning the function M, we assume that $M\in C^1(\mathbb{R})$ and there exists $\gamma>0$ satisfying

$$0\le\hat{M}(s)\le\gamma M(s)s\quad\text{for } s\ge0, \tag{2.1}$$

where $\hat{M}(s)=\int_0^s M(\tau)d\tau$. For the relaxation function g and potential σ, as in [20], we assume that $g,\ \sigma:\mathbb{R}_+\to\mathbb{R}_+$ are nonincreasing differentiable functions satisfying

$$g(0)>0,\qquad l_0:=\int_0^\infty g(s)ds<\infty,\qquad \sigma(t)>0,\qquad 1-\frac{2\sigma(t)}{\lambda_1}\int_0^t g(s)ds\ge l>0\quad\text{for } t\ge0, \tag{2.2}$$

and there exists a nonincreasing differentiable function $\zeta:\mathbb{R}_+\to\mathbb{R}_+$ with

$$\zeta(t)>0,\qquad g'(t)\le-\zeta(t)g(t)\quad\text{for } t\ge0,\qquad \lim_{t\to\infty}\frac{-\sigma'(t)}{\zeta(t)\sigma(t)}=0. \tag{2.3}$$