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Delay-independent stability of homogeneous systems

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ABSTRACT

A class of nonlinear systems with homogeneous right-hand sides and time-varying delay is studied. It is assumed that the trivial solution of a system is asymptotically stable when delay is equal to zero. By the usage of the Lyapunov direct method and the Razumikhin approach, it is proved that the asymptotic stability of the zero solution of the system is preserved for an arbitrary continuous nonnegative and bounded delay. The conditions of stability of time-delay systems by homogeneous approximation are obtained. Furthermore, it is shown that the presented approaches permit to derive delay-independent stability conditions for some types of nonlinear systems with distributed delay. Two examples of nonlinear oscillatory systems are given to demonstrate the effectiveness of our results.

1. Introduction

Stability of nonlinear time-delay systems has attracted an increasing attention during last decades, mainly due to the numerous applications of these systems in engineering, mechanics, life sciences, chemistry and economics, see, e.g., [1–6]. There are two principal approaches to the stability analysis of time-delay systems, the first one is based on the Lyapunov–Krasovskii functionals [1,2], the other one is based on the Razumikhin theorem [2,3]. These approaches were successfully applied by various authors to the study of wide classes of systems, see [1–10] and the references cited therein.

Stability conditions may depend on delay, and in this case, we cannot guarantee that the system remains stable when delay exceeds a certain value. In some applications, it is not possible to ensure that delay is sufficiently small, and even known. Therefore, it is important to have stability conditions under which the system remains stable for any positive value of delay. Such conditions are known as delay-independent ones [1,3,6,7]. For example, some applications of the conditions appear in the analysis of population dynamics [4,5]. Several delay-independent stability conditions for various classes of nonlinear systems can be found in [1–3,6,10–14]. In particular, interesting delay-independent stability conditions based on a modification of the Razumikhin approach have been derived in [11,13], homogeneous and subhomogeneous cooperative systems have been studied in [12,15], the case of bilinear homogeneous systems has been studied in [14]. The problem of delay-independent stability is especially difficult for systems with time-varying and distributed delay [2,3,8,10].

It is known, that Lyapunov functions constructed for delay free systems can be used for the stability analysis of time-delay systems [1–3]. In [16,17], this idea has been combined with the Razumikhin approach in order to derive delay-independent stability conditions for some classes of nonlinear time-delay systems.

In this paper, we extend results obtained in [16,17] to more general classes of systems. We study homogeneous systems with time-varying delay. It is assumed that the trivial solutions of considered systems are asymptotically stable when delay is equal to zero. By the usage of the Lyapunov direct method and the Razumikhin approach, it is proved that if the degree of

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homogeneity of a system is positive, then the asymptotic stability of the zero solution is preserved for an arbitrary continuous nonnegative and bounded delay. The conditions of stability of time-delay systems by homogeneous approximation are obtained. Furthermore, it is shown that the presented approaches permit to derive delay-independent stability conditions for some types of nonlinear systems with distributed delay. Two examples of nonlinear oscillatory systems with time-varying delay are given to demonstrate the effectiveness of our results.

2. Preliminaries

In the sequel, $\mathbb{R}$ denotes the field of real numbers, and $\mathbb{R}^n$ the n-dimensional Euclidean space. For a given real number $h \in (0, +\infty)$, let $C([-h, 0], \mathbb{R}^n)$ be the space of continuous functions $\varphi(\theta) : [-h, 0] \to \mathbb{R}^n$. The Euclidean norm will be used for vectors. For elements of the space $C([-h, 0], \mathbb{R}^n)$ we will use the uniform (supremum) norm $\|\varphi\|_h = \max_{\theta\in[-h,0]} \|\varphi(\theta)\|$.

In this paper, time-delay systems of the form

$$\dot{\mathbf{x}}(t) = \mathbf{G}(t, \mathbf{x}_t), \quad t \geq 0, \tag{1}$$

are studied. Here $\mathbf{x}(t) \in \mathbb{R}^n$ is the state vector. It is assumed that the functional $\mathbf{G}(t, \varphi)$ is continuous in the domain

$$\{t \in \mathbb{R} : t \geq 0\} \times \Omega_H, \tag{2}$$

where Ω_H is the set of functions $\varphi(\theta) \in C([-h, 0], \mathbb{R}^n)$ satisfying the inequality $\|\varphi\|_h < H$, $0 < H \leq +\infty$. Let $\mathbf{x}(t, t_0, \varphi)$ stand for a solution of system (1) with the initial conditions $t_0 \geq 0$, $\varphi(\theta) \in \Omega_H$, and $\mathbf{x}_t(t_0, \varphi)$ denote the restriction of the solution to the segment $[t-h, t]$, i.e., $\mathbf{x}_t(t_0, \varphi) : \theta \to \mathbf{x}(t+\theta, t_0, \varphi)$, $\theta \in [-h, 0]$. In some cases, when the initial conditions are not important, or are well defined from the context, we write $\mathbf{x}(t)$ and $\mathbf{x}_t$, instead of $\mathbf{x}(t, t_0, \varphi)$ and $\mathbf{x}_t(t_0, \varphi)$, respectively.

Next, let us introduce the concept of homogeneity [18,19] for the following analysis.

Definition 1. Let $\mathbf{x} = (x_1, \ldots, x_n)^T$. A function $V(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}$ is called homogeneous of degree $\mu \in \mathbb{R}$ with respect to the dilation or weights $(m_1, \ldots, m_n)$, where $m_i > 0$, $i = 1, \ldots, n$, if

$$V(\lambda^{m_1}x_1, \ldots, \lambda^{m_n}x_n) = \lambda^{\mu} V(x_1, \ldots, x_n)$$

for all $\lambda > 0$ and $\mathbf{x} \in \mathbb{R}^n$. A vector field $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \ldots, f_n(\mathbf{x}))^T : \mathbb{R}^n \to \mathbb{R}^n$ is called homogeneous of degree $\mu \in \mathbb{R}$ with respect to weights $(m_1, \ldots, m_n)$, where $m_i > 0$, $i = 1, \ldots, n$, if

$$f_i(\lambda^{m_1}x_1, \ldots, \lambda^{m_n}x_n) = \lambda^{\mu+m_i} f_i(x_1, \ldots, x_n), \quad i = 1, \ldots, n,$$

for all $\lambda > 0$ and $\mathbf{x} \in \mathbb{R}^n$. The system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t))$ is called homogeneous if its vector field $\mathbf{f}(\mathbf{x})$ is homogeneous.

Remark 1. In the case when $V(\lambda\mathbf{x}) = \lambda^{\mu} V(\mathbf{x})$ for all $\lambda > 0$ and $\mathbf{x} \in \mathbb{R}^n$, the function $V(\mathbf{x})$ is called homogeneous of degree μ with respect to the standard dilation.

In a similar way the concept of homogeneity for vector functionals is introduced, see [13].

Definition 2. A vector functional

$$\mathbf{f}(\varphi) = (f_1(\varphi), \ldots, f_n(\varphi))^T : C([-h, 0], \mathbb{R}^n) \to \mathbb{R}^n$$

is called homogeneous of degree $\mu \in \mathbb{R}$ with respect to weights $(m_1, \ldots, m_n)$, where $\varphi = (\varphi_1, \ldots, \varphi)^T$, and $m_i > 0$, $i = 1, \ldots, n$, if

$$f_i(\lambda^{m_1}\varphi_1, \ldots, \lambda^{m_n}\varphi_n) = \lambda^{\mu+m_i} f_i(\varphi_1, \ldots, \varphi_n), \quad i = 1, \ldots, n,$$

for all $\lambda > 0$ and $\varphi \in C([-h, 0], \mathbb{R}^n)$. A time-delay system $\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}_t)$ is homogeneous when $\mathbf{f}$ is homogeneous.

3. Statement of the problem

Consider a system of differential equations with delay of the form

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t - \tau(t))) \tag{3}$$

and the corresponding delay free system

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{x}(t)). \tag{4}$$

Here $\mathbf{x}(t) \in \mathbb{R}^n$; vector function $\mathbf{f}(\mathbf{x}, \mathbf{y})$ is defined for all $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$; function $\tau(t)$ is continuous nonnegative and bounded for $t \geq 0$.

Assumption 1. The function $\mathbf{f}(\mathbf{x}, \mathbf{y})$ is continuous for $\mathbf{x}, \mathbf{y} \in \mathbb{R}^n$ and homogeneous of degree $\mu \in \mathbb{R}$ with respect to weights $(m_1, \ldots, m_n, m_1, \ldots, m_n)$, where $m_i > 0$, $\mu + m_i > 0$, $i = 1, \ldots, n$.

If Assumption 1 is fulfilled, then systems (3) and (4) admit the zero solutions.