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An explicit criterion for finite-time stability of linear nonautonomous systems with delays

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ABSTRACT

In this paper, the problem of finite-time stability of linear nonautonomous systems with time-varying delays is considered. Using a novel approach based on some techniques developed for linear positive systems, we derive new explicit conditions in terms of matrix inequalities ensuring that the state trajectories of the system do not exceed a certain threshold over a pre-specified finite time interval. These conditions are shown to be relaxed for the Lyapunov asymptotic stability. A numerical example is given to illustrate the effectiveness of the obtained result.

1. Introduction

The stability of time delay systems has been one of the most attractive research topics during the past decades [1–5]. While the concept of Lyapunov stability, recognized as infinite time behavior, has been well investigated and developed, the concept of finite-time stability (FTS) (or short-time stability) has been extensively studied in recent years (see, [6–14] and the references therein).

Roughly speaking, a system is finite-time stable if, for a given bound on the initial condition, its state trajectories do not exceed a certain threshold during a pre-specified time interval [7]. It is noted that, a system may be finite-time stable but not Lyapunov asymptotic stable, and vice versa [6,7,13] (see, also, Remark 2.1 in this paper). Although the Lyapunov asymptotic stability (LAS) has been successfully applied in many models, FTS is a useful concept to study in many practical systems in the vivid world [8–10,12,13].

However, most of the existing results in the literature so far have been devoted to linear autonomous (i.e. time-invariant) systems. For linear time-invariant systems with constant delay, some finite-time stability conditions have been derived in terms of feasible linear matrix inequalities based on the main approach is the Lyapunov–Krasovskii functional method [6,10–14]. There has been no result concerned with the FTS of nonautonomous systems (time-varying systems) with time-varying delays. Moreover, it should be noted that, the proposed conditions for FTS of time-varying systems based on the Lyapunov functional approach have usually been derived in terms of Lyapunov or Riccati matrix differential equations [7–9] which lead to indefinite matrix inequalities with lack of efficient computational tools to solve them. Therefore, an alternative approach when dealing with the FTS of time-varying systems with delays is clearly needed, which has motivated our present investigation.

In this paper, we consider the problem of FTS of linear nonautonomous systems with discrete and distributed time-varying delays. By utilizing some techniques developed for linear positive systems, we derive new explicit conditions in

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terms of matrix inequalities ensuring that, for each given bound on the initial conditions, the state trajectories of the system do not exceed a certain threshold over a pre-specified finite time interval. Our conditions are derived in terms of some types of the Metzler matrix which can be easily verified. The novel feature of the result obtained in this paper is twofold. Firstly, the system considered in this paper is time-varying subjected to interval, nondifferentiable delays, which means that the lower and the upper bounds for the time-varying delays are available but the delay functions are not necessary to be differentiable. This allows that the time-delays can be fast time-varying functions. Secondly, by a novel approach without using the Lyapunov–Krasovskii functional method, we derive an explicit criterion for the FTS of the system in terms of Metzler matrix inequalities which is intuitive and easy to verify.

Notations. For a given positive integer n, we denote $\underline{n} := \{1, 2, \dots, n\}$. R^n denotes the n-dimensional space with the norm $\|x\|_\infty = \max_{i\in\underline{n}} |x_i|$. The set of real $m \times n$-matrices is denoted by $R^{m\times n}$. For $u = (u_i)$, $v = (v_i)$ in R^n, $u \geq v$ iff $u_i \geq v_i$, $\forall i \in \underline{n}$; $u \gg v$ iff $u_i > v_i$, $\forall i \in \underline{n}$. We denote a vector $e = (1 \dots 1)^T \in R^n$.

2. Problem statement and preliminaries

Consider the following linear nonautonomous system with time-varying delays

$$\begin{aligned} &\dot{x}(t) = A(t)x(t) + D(t)x(t-\tau(t)) + G(t)\int_{t-\kappa(t)}^{t} x(s)ds, \quad t \geq 0, \\ &x(t) = \phi(t), \quad t \in [-d, 0], \end{aligned} \tag{2.1}$$

where $x(t) \in R^n$ is the state; $A(t) = (a_{ij}(t)) \in R^{n\times n}$, $D(t) = (d_{ij}(t)) \in R^{n\times n}$ and $G(t) = (g_{ij}(t)) \in R^{n\times n}$ are the system matrices; $\tau(t), \kappa(t)$ are time-varying delays satisfying $0 \leq \underline{\tau} \leq \tau(t) \leq \overline{\tau}$, $0 \leq \kappa(t) \leq \overline{\kappa}$, $t \geq 0$; $\phi(t) = (\phi_i(t)) \in C([-d, 0], R^n)$, where $d = \max\{\overline{\tau}, \overline{\kappa}\}$, is the initial condition. Let us denote $|\phi_i| = \sup_{-d\leq t\leq 0} |\phi_i(t)|$ and $\|\phi\|_\infty = \max_{i\in\underline{n}} |\phi_i|$.

Definition 2.1. For given a time $T > 0$ and positive numbers $r_1 < r_2$, system (2.1) is said to be finite-time stable with respect to (r_1, r_2, T) if for any initial condition $\phi(t) \in C([-d, 0], R^n)$, $\|\phi\|_\infty \leq r_1$ implies that $\|x(t, \phi)\|_\infty < r_2$ for all $t \in [0, T]$.

Remark 2.1. It should be noted that, FTS and LAS are independent concepts in the following sense: a system which is FTS may be not LAS, and vice versa. This will be illustrated in the following example.

Example 2.1. Consider the following delay differential equations

$$\dot{x}(t) = -1.2x(t) + \frac{t+2}{t+1}x(t-1), \quad t \geq 0, \tag{2.2}$$

$$\dot{x}(t) = -0.8x(t) + \frac{t}{t+6}x(t-1), \quad t \geq 0. \tag{2.3}$$

Eq. (2.2) is globally LAS. However, this equation is not FTS with respect to $r_1 = 1$, $r_2 = 1.25$ and $T = 10$. Conversely, Eq. (2.3) is FTS with respect to $r_1 = 1$, $r_2 = 1.5$ and $T = 10$ but (2.3) is not LAS, even every non-zero solution of (2.3) goes to infinity as time tends to infinity. The state trajectories of (2.2) and (2.3) with initial condition $\phi(t) = 1$, $t \in [-1, 0]$, are presented in Figs. 1 and 2, respectively.

The main purpose of this paper is to find conditions for the stability of system (2.1) over a finite time interval $[0, T]$. By utilizing some techniques developed for positive systems, some new explicit conditions are derived in terms of the Metzler matrix for the FTS of system (2.1).

3. Main results

Let $A(t) = (a_{ij}(t))$, $D(t) = (d_{ij}(t))$ and $G(t) = (g_{ij}(t))$ be given matrices with continuous elements. We make the following assumptions which are usually used for time-varying systems (see, for example, [3]). For given $T > 0$, assume that

A1. $a_{ii}(t) \leq \overline{a}_{ii}$, $i \in \underline{n}$, $|a_{ij}(t)| \leq \overline{a}_{ij}$, $i \neq j$, $i, j \in \underline{n}$, $t \in [0, T]$.

A2. $|d_{ij}(t)| \leq \overline{d}_{ij}$, $|g_{ij}(t)| \leq \overline{g}_{ij}$, $t \in [0, T]$, $i, j \in \underline{n}$.

We denote $\mathcal{A} = (\overline{a}_{ij})$, $\mathcal{D} = (\overline{d}_{ij})$ and $\mathcal{G} = (\overline{g}_{ij})$. For a nonnegative scalar γ, let us define the matrix $\mathcal{M}_\gamma = \mathcal{A} - \gamma I + e^{-\gamma\underline{\tau}}\mathcal{D} + \overline{\kappa}\mathcal{G}$.