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On the new results of global exponential attractive set

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ABSTRACT

In this paper, the global exponential attractive sets of a class of continuous-time dynamical systems defined by $\dot{x} = f(x)$, $x \in R^n$ are studied. The elements of main diagonal of matrix A are both negative numbers and zero, where matrix A is the Jacobian matrix $\frac{df}{dx}$ of a continuous-time dynamical system defined by $\dot{x} = f(x)$, $x \in R^n$ evaluated at the origin $x_0 = (0, 0, \ldots, 0)_{1\times n}$. However, note that the former equations that we are searching for a global bounded region have a common characteristic: the elements of main diagonal of matrix A are all negative. As far as we know, very few papers have addressed this problem.

1. Introduction

A chaotic system is bounded in the sense that its chaotic attractor is bounded in the phase space. The bounds of the Lorenz system were first studied by G.A. Leonov [1,2]. Then, Liao et al. obtained the new global attractive and positive invariant sets of the Lorenz system by constructing a family of generalized Lyapunov functions [3]. The globally exponential attractive set of a chaotic system plays an important role in chaos control, chaos synchronization, and many other applications. If one can show that a chaotic or a hyperchaotic system has a globally attractive set, then the system cannot possess hidden attractors outside the globally attractive set. This is very important for engineering applications [4,5], since it is very difficult to predict the existence of hidden attractors and they can lead to crashes. In searching for a global bounded region, one generally would like to choose a Lyapunov function, as simple as possible, and apply the Lyapunov stability criteria. However, note that the former equations that we are searching for a global bounded region have a common characteristic: the elements of main diagonal of matrix A are all negative [6–11], where matrix A is the Jacobian matrix $\frac{df}{dx}$ of a continuous-time dynamical system defined by $\dot{x} = f(x)$, $x \in R^n$ evaluated at the origin. And the Lyapunov functions for the 3D chaotic systems (The method of constructing the Lyapunov functions for the high-dimensional chaotic systems is similar) in [6–11] can be constructed in the form as $V(x, y, z) = a_1 (x - a_2)^2 + b_1 (y - b_2)^2 + c_1 (y - c_2)^2$, $a_1 > 0, b_1 > 0, c_1 > 0, a_2, b_2, c_2 \in R$. However, for the reason that the elements of main diagonal of matrix A are both negative and zero for system (1), it seems that a quadratic Lyapunov function is not sufficient for this purpose, which is quite different from the systems in [6–11]. Therefore, the approach for constructing the Lyapunov functions that applied to the systems in [3,6–11] does not work for chaotic system (1). We overcome this difficulty by introducing a cross term xy and construct Lyapunov functions for system (1) as $V_{m,n}(x, y, z) = mx^2 + (y - nx)^2 + a(z - \tau)^2$, $\forall m > 0, 0 < \forall n < \frac{c}{3a}, \tau = m + n + b - n^2 > 0$. The global exponential attractive sets for the Lorenz-like system (1) are obtained in this paper through many proper inequalities and the proper Lyapunov functions.

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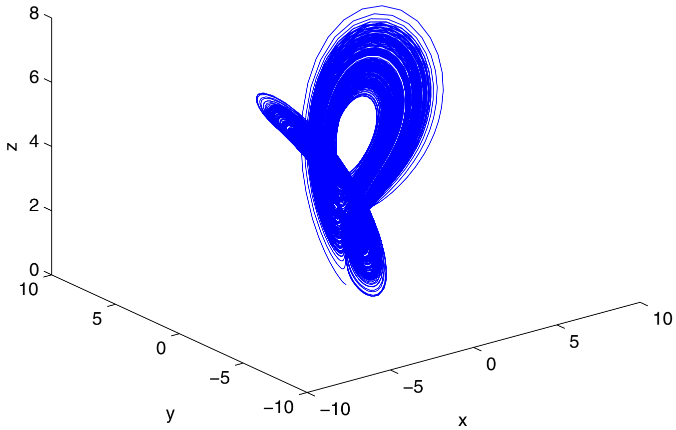

Fig. 1. Phase portrait of system (1) in the x–y–z space.

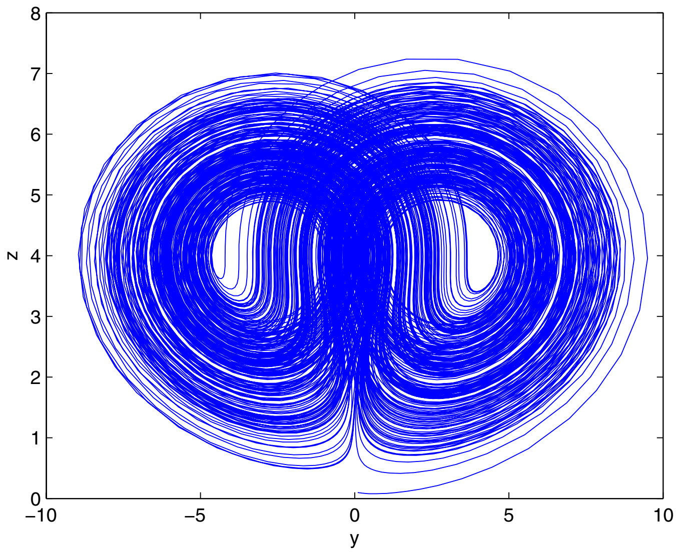

Fig. 2. Phase portrait of system (1) in the y–z plane.

2. Mathematical model

Recently, Li et al. has introduced a Lorenz-like chaotic system [12]:

$$\begin{cases} \dot{x} = -ax + ay \\ \dot{y} = abx - axz \\ \dot{z} = xy - cz \end{cases} \tag{1}$$

where $a > 0, b > 0, c > 0$ are parameters. When $a = 5, b = 4, c = 2$, system (1) is chaotic [12] and phase portraits of system (1) are shown in Figs. 1 and 2. Ref. [12] has proved that system (1) is not topologically equivalent to the Chen system, the Lu system, the unified system, the Liu system [13] and other known Lorenz-like systems.

3. Main results

In the following, we will discuss the global exponential attractive sets of system (1) for the case of $2a > c > 0, b > 0$ (in system (1) $a = 5, b = 4, c = 2$). First, we introduce the following Lemma.