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Effect of off-diagonal delay on the asymptotic stability for an integro-differential system

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Dedicated to Professor Satoru Murakami on the occasion of his 60th birthday

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A B S T R A C T

Our concern is to solve the stability problem for a linear integro-differential system with distributed delay in the off-diagonal terms. Some new necessary and sufficient conditions are established for the zero solution of the system to be asymptotically stable. The proof of our main theorem is given by a careful analysis of the locations of roots of the associated characteristic equation.

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1. Introduction

The stability of the trivial solution of delay differential systems depends on the time delays from the feedback as well as the parameters describing the models. Over the past few decades, many articles have been devoted to the study of the delay effect on the stability. For example, the results for linear autonomous systems can be found in [1–7].

In this paper we are concerned with the stability problem for a linear integro-differential system of the form

$$\begin{cases} x'(t) = -ax(t) - b\displaystyle\int_{t-r}^{t} y(s)ds, \\ y'(t) = -c\displaystyle\int_{t-r}^{t} x(s)ds - ay(t), \end{cases} \tag{1.1}$$

where a, b and c are real numbers and r is a positive number. It is well known (see, e.g., [8]) that for linear autonomous differential systems with delay, the asymptotic stability of the trivial solution (i.e., the zero solution) is equivalent to all solutions having limit zero as $t \to \infty$ which in turn is true if and only if all roots of an associated characteristic equation have negative real parts.

System (1.1) is a special case of the delay system

$$\begin{cases} x'(t) = -a_1x(t) - b_1\displaystyle\int_{-\infty}^{t} K_1(t-s)y(s)ds, \\ y'(t) = -b_2\displaystyle\int_{-\infty}^{t} K_2(t-s)x(s)ds - a_2y(t), \end{cases} \tag{1.2}$$

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where, for $j = 1, 2$, a_j and b_j are real numbers and K_j are delay kernels defined and integrable on $[0, \infty)$. System (1.2) appears as the linearization of population models of Lotka–Volterra type; see [3,4,9,10] and references therein.

When $K_j(t-s) = \delta(t-s-r_j)$, where δ denotes the delta function and r_1 and r_2 are positive numbers, system (1.2) becomes the differential-difference system

$$\begin{cases} x'(t) = -a_1x(t) - b_1y(t-r_1), \\ y'(t) = -b_2x(t-r_2) - a_2y(t). \end{cases} \tag{1.3}$$

Recently, the stability problem for system (1.3) with discrete delay has been solved by Wei and Zhang [7] and the present authors [6] independently. However, there are very few results on the asymptotic stability of system (1.1) with distributed delay.

The purpose of this paper is to establish necessary and sufficient conditions for the zero solution of (1.1) to be asymptotically stable. The following theorem is our main result.

Theorem 1.1. *The zero solution of (1.1) is asymptotically stable if and only if any one of the following three conditions holds:*

$$a > 0, \qquad bc > 0 \quad \text{and} \quad r < \frac{a}{\sqrt{bc}}, \tag{1.4}$$

$$a > 0 \quad \text{and} \quad bc = 0, \tag{1.5}$$

$$a > 0, \qquad bc < 0 \quad \text{and} \quad r < -\frac{1}{\omega_0} \arccos\left(\frac{a\omega_0}{\sqrt{-bc}} + 1\right), \tag{1.6}$$

where ω_0 is the unique negative root of the cubic equation $\omega^3 + a^2\omega + 2a\sqrt{-bc} = 0$ with $a > 0$ and $bc < 0$.

Remark 1.1. In the case $bc = 0$, system (1.1) is reduced to a scalar differential equation $x'(t) = -ax(t)$, and, thus, one can immediately conclude that condition (1.5) is the necessary and sufficient condition for the asymptotic stability of (1.1) with $bc = 0$.

Remark 1.2. Condition (1.4), (1.5), or (1.6) is equivalent to

$$a > 0 \quad \text{and} \quad -\left(\frac{a\omega_0}{1 - \cos \omega_0 r}\right)^2 < bc < \frac{a^2}{r^2},$$

which is another explicit condition for the asymptotic stability of (1.1).

The rest of this paper is organized as follows. In Section 2, we introduce some auxiliary results on the locations of roots of transcendental equations which will be used in our proofs. In Section 3, we prove our main theorem.

2. Preliminaries

We consider a transcendental equation

$$\lambda + p + q \int_{-r}^{0} e^{\lambda s} ds = 0, \tag{2.1}$$

where p and q are real numbers and r is a positive number. Eq. (2.1) is the characteristic equation of a scalar integro-differential equation

$$x'(t) = -px(t) - q \int_{t-r}^{t} x(s)ds.$$

The locations of roots of (2.1) have been studied in [2,11,12]. Recently, Hara and Sakata [13] proved the following result which was conjectured by Funakubo et al. [2].

Theorem A (*[2,13]*). *All roots of (2.1) lie in the left half of the complex plane if and only if any one of the following four conditions holds:*

(i) $p > 0, \quad q \geq 0 \quad \text{and} \quad 2q - p^2 \leq 0,$

(ii) $p \geq 0, \quad q > 0, \quad 2q - p^2 > 0 \quad \text{and} \quad r < \dfrac{1}{\sqrt{2q - p^2}}\left(2\pi - \arccos \dfrac{p^2 - q}{q}\right),$

(iii) $p > 0, \quad q < 0 \quad \text{and} \quad r < \left|\dfrac{p}{q}\right|,$

(iv) $p < 0, \quad q > 0, \quad 2q - p^2 > 0 \quad \text{and} \quad \left|\dfrac{p}{q}\right| < r < \dfrac{1}{\sqrt{2q - p^2}} \arccos \dfrac{p^2 - q}{q}.$

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