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Combined bracketing methods for solving nonlinear equations

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ABSTRACT

Several methods based on combinations of bisection, regula falsi, and parabolic interpolation has been developed. An interval bracketing ensures the global convergence while the combination with the parabolic interpolation increases the speed of the convergence. The proposed methods have been tested on a series of examples published in the literature and show good results.

1. Introduction

Many bracketing algorithms have been described in literature for finding a single root of a nonlinear equation

$$f(x) = 0. \tag{1}$$

The most basic bracketing method is a dichotomy method also known as a bisection method with a rather slow convergence [1]. The method is guaranteed to converge for a continuous function f on the interval $[x_a, x_b]$ where $f(x_a)f(x_b) < 0$. Another basic bracketing root finding method is a regula falsi technique (false position) [1]. Both methods have a linear order of convergence, but the regula falsi method suffers due to the slow convergence in some cases. A variant of the regula falsi method called an Illinois algorithm improves this drawback [2]. In order to increase reliability, combinations of several methods were introduced by Dekker [3] who combined the bisection and secant method. This method was further improved by Brent [4] and later by Alefeld and Potra [5] with several new algorithms. An approximation with an exponential function was also used on the closed interval in order to calculate a root of the nonlinear function [6].

Instead of a linear interpolation, a quadratic polynomial interpolation can be also used for the root determination. This open technique with a superlinear order of convergence was introduced by Muller [7] and successfully applied on the closed interval by Suhadolnik [8]. Muller's method can also be found in the combination with the bisection [9–11] or inverse quadratic interpolation [12]. Recently, several new algorithms of enclosing methods were published [13–18].

2. Proposed zero finding methods

In the rest of the text, it will be assumed that the function $f(x)$ is continuous on a closed interval $[x_a, x_b]$. In this interval, the function has a root and the following inequality holds

$$f(x_a)f(x_b) < 0. \tag{2}$$

Without loss of generality we suppose that the function values on the border of the interval are $f(x_a) < 0$ and $f(x_b) > 0$. A new iterative value on the closed interval is calculated by fitting a parabola to the three points of the function $f(x)$. The first

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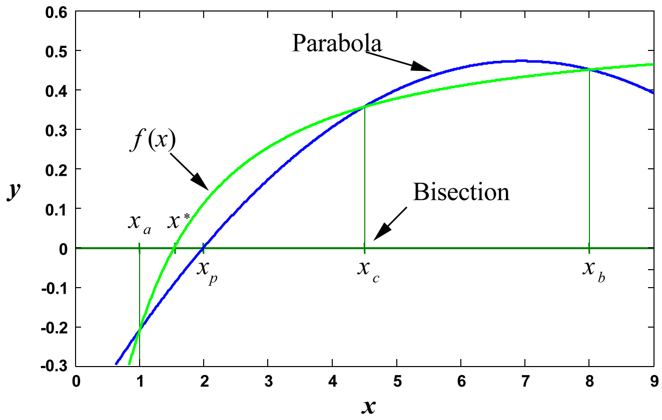

Fig. 1. Bisection-parabolic method (BP).

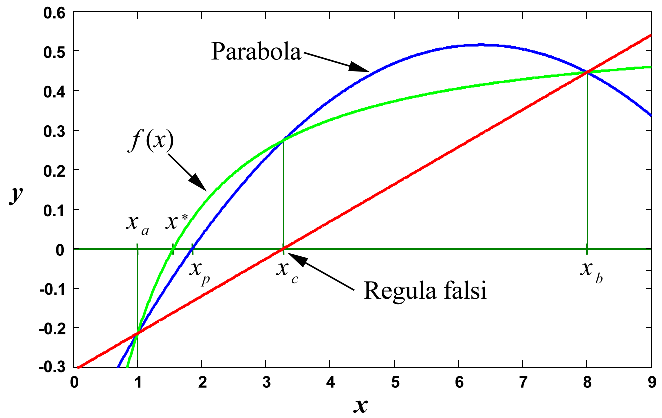

Fig. 2. Regula falsi–parabolic method (RP).

and second points are the interval border points $(x_a, f(x_a))$ and $(x_b, f(x_b))$ while the third point $(x_c, f(x_c))$; $x_c \in (x_a, x_b)$ is calculated by using the bisection (Fig. 1)

$$x_c = \frac{x_a + x_b}{2}, \tag{3}$$

or regula falsi algorithm (Fig. 2)

$$x_c = \frac{f(x_a)x_b - f(x_b)x_a}{f(x_a) - f(x_b)}. \tag{4}$$

We call the first method a bisection–parabolic (BP) method and the second a regula falsi–parabolic (RP) method. Unfortunately, in some cases the regula falsi method has a rather slow convergence (see examples in Table 1) which can remain even in the combined regula falsi–parabolic method. In order to prevent slow convergence in such cases a switching between the bisection and regula falsi is used to calculate the third point x_c. This method is called a regula falsi–bisection–parabolic (RBP) method.

The switching mechanism between the regula falsi and bisection bases on the observation of an interpolating line slope. If the slope is too high or too low the bisection is engaged instead of the regula falsi. This procedure prevents slow convergence in some cases when only the RP method is used (see examples in Table 1 for comparison) and exploits cases where the RP method is faster than the BP method.

After calculating x_c, three points $(x_a, f(x_a))$, $(x_c, f(x_c))$ and $(x_b, f(x_b))$ are finally available and through these points, a second order polynomial can be constructed

$$p(x) = A(x - x_c)^2 + B(x - x_c) + C. \tag{5}$$

The complete polynomial equations for all three points are then

$$\begin{cases} f(x_a) = A(x_a - x_c)^2 + B(x_a - x_c) + C, \\ f(x_b) = A(x_b - x_c)^2 + B(x_b - x_c) + C, \\ f(x_c) = C. \end{cases} \tag{6}$$