Applied Mathematics Letters 25 (2012) 555–560

Contents lists available at SciVerse ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

Existence and uniqueness of positive solutions for higher order nonlocal fractional differential equations

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ARTICLE INFO

Article history:
Received 30 April 2011
Received in revised form 21 September 2011
Accepted 26 September 2011

Keywords:
Monotone iterative technique
Fractional differential equation
Existence and uniqueness
Positive solution

ABSTRACT

By means of a monotone iterative technique, we establish the existence and uniqueness of the positive solutions for a class of higher conjugate-type fractional differential equation with one nonlocal term. In addition, the iterative sequences of solution and error estimation are also given. In particular, this model comes from economics, financial mathematics and other applied sciences, since the initial value of the iterative sequence can begin from an known function, this is simpler and helpful for computation.

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1. Introduction

In this paper, we are concerned with the existence and uniqueness of positive solutions for the following singular nonlinear $(n-1, 1)$ conjugate-type fractional differential equation with one nonlocal term

$$\begin{cases} D^{\alpha}_{0+}x(t) + f(t, x(t)) = 0, \quad 0 < t < 1, \ n-1 < \alpha \le n, \\ x^{(k)}(0) = 0, \quad 0 \le k \le n-2, \qquad x(1) = \displaystyle\int_0^1 x(s)dA(s), \end{cases} \tag{1.1}$$

where $\alpha \ge 2$, D^{α}_{0+} is the standard Riemann–Liouville derivative, A is a function of bounded variation and $\int_0^1 u(s)dA(s)$ denotes the Riemann–Stieltjes integral of u with respect to A, dA can be a signed measure.

Since $\int_0^1 u(s)dA(s)$ denotes the Riemann–Stieltjes integral in BCs (1.1), this implies the case of BCs (1.1) covers the multi-point BCs and also integral BCs in a single framework. For a comprehensive study of the case when there is a Riemann–Stieltjes integral boundary condition at both ends, see [1].

As the boundary value problem in economics, financial mathematics and other applied science has a wide range of applications, in recent years, there have been many papers investigating the existence and uniqueness of the positive solution for local or nonlocal boundary value problems of the second or higher order ordinary differential equations, we refer the readers to [2–9] and the references cited therein. For the case where α is an integer, Du and Zhao [9] investigated the following multi-point boundary problem

$$\begin{cases} -x''(t) = f(t, x(t)), \quad 0 < t < 1, \\ x(0) = \displaystyle\sum_{i=1}^{m-2} \alpha_i x(\eta_i), \qquad x(1) = 0. \end{cases} \tag{1.2}$$

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doi:10.1016/j.aml.2011.09.058

They assumed f is decreasing in u and obtained the existence of $C[0,1]$ positive solutions w for (1.2) with the property that $w(t) \geq m(1-t)$ for some $m > 0$. In a recent paper [10], Webb and Zima studied the problem

$$\begin{cases} x''(t) + k^2 x = f(t, x(t)), \quad 0 < t < 1, \\ x(0) = 0, \qquad x(1) = \displaystyle\int_0^1 x(s)dA(s) \end{cases} \tag{1.3}$$

when dA is allowed to be a signed measure, and obtained the existence of multiple positive solutions under suitable conditions on $f(t,x)$. And then, by applying the monotone iterative technique, Mao et al. [8] established the existence and uniqueness of the positive solution for singular integral boundary value problem (1.3). When α is a fraction, Goodrich [11] dealt with a problem similar to (1.1) but with local conditions, by deriving properties of the Green's function and by using the well-known Guo–Krasnoselskii's fixed point theorem, the author established some nice existence results of at least one positive solution provided that $f(t,x)$ satisfies some growth conditions. Similarly, a significant work is developed by Goodrich [12] to study another fractional problem of nonlocal-type similar to (1.1) by utilizing different techniques from [11] and here. Recently, the same problem (1.1) is treated by Wang et al. [13] through cone theoretic techniques, where $f(t,x)$ can be singular at $x = 0$. Their techniques are also rather different from the ones presented here.

We have found that until now no result has been established for the existence and uniqueness of positive solutions for the problem (1.1) of a fractional differential equation when f has singularities at $t = 0$ and (or) 1. This paper thus aims to establish the existence and uniqueness of positive solutions for the problem (1.1), moreover we also obtain error estimates and the convergence rate of positive solutions with the property that there exist $M > m > 0$ such that $mt^{\alpha-1} \leq w^* \leq Mt^{\alpha-1}$.

2. Preliminaries and lemmas

For the convenience of the reader, we present here the Riemann–Liouville definitions for the fractional integral and derivative from fractional calculus which are to be used in the later sections.

Definition 2.1 (*See [14]*). Let $\alpha > 0$ with $\alpha \in \mathbb{R}$. Suppose that $x : [a, \infty) \to \mathbb{R}$. Then the αth Riemann–Liouville fractional integral is defined to be

$$D_{0+}^{-\alpha} x(t) = \frac{1}{\Gamma(\alpha)} \int_a^t (t-s)^{\alpha-1} x(s) ds$$

whenever the right-hand side is defined. Similarly, with $\alpha > 0$ with $\alpha \in \mathbb{R}$, we define the αth Riemann–Liouville fractional derivative to be

$$D_{0+}^{\alpha} x(t) = \frac{1}{\Gamma(n-\alpha)} \left(\frac{d}{dt}\right)^{(n)} \int_a^t (t-s)^{n-\alpha-1} x(s) ds,$$

where $n \in \mathbb{N}$ is the unique positive integer satisfying $n - 1 \leq \alpha < n$ and $t > a$.

Proposition 2.1 (*See [15,14]*). *Let $\alpha > 0$, and $f(x)$ is integrable, then*

$$D_{0+}^{-\alpha} D_{0+}^{\alpha} f(x) = f(x) + c_1 x^{\alpha-1} + c_2 x^{\alpha-2} + \cdots + c_n x^{\alpha-n}$$

where $c_i \in \mathbb{R}$ $(i = 1, 2, \ldots, n)$, n is the smallest integer greater than or equal to α.

Proposition 2.2 (*See [15,14]*). *The equality*

$$D_{0+}^{\alpha} D_{0+}^{-\alpha} f(x) = f(x), \quad \alpha > 0$$

holds for $f \in L^1(a, b)$.

Lemma 2.1 (*See [16]*). *Given $y \in L^1(0,1)$, then the problem*

$$\begin{cases} D_{0+}^{\alpha} x(t) + y(t) = 0, \quad 0 < t < 1, \\ x(0) = x'(0) = \cdots = x^{(n-2)} = 0, \qquad x(1) = 0, \end{cases} \tag{2.1}$$

has the unique solution

$$x(t) = \int_0^1 G(t,s) y(s) ds, \tag{2.2}$$

where $G(t,s)$ is the Green function of BCs (2.1) and is given by

$$G(t,s) = \frac{1}{\Gamma(\alpha)} \begin{cases} [t(1-s)]^{\alpha-1}, & 0 \leq t \leq s \leq 1, \\ [t(1-s)]^{\alpha-1} - (t-s)^{\alpha-1}, & 0 \leq s \leq t \leq 1. \end{cases} \tag{2.3}$$

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