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The optimal decay rate for a weak dissipative Bresse system

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ABSTRACT

In this work we consider the Bresse system with frictional damping operating only on the angle displacement and we show that under a certain assertion the solution decays polynomially and the decay rate is optimal.

1. Introduction

In this work, we consider a Bresse system with frictional damping effective only in one equation of the system. The system is written as

$$\rho_1\varphi_{tt} - k(\varphi_x + \psi + lw)_x - k_0 l(w_x - l\varphi) = 0 \quad \text{in } (0,\infty)\times(0,L) \tag{1.1}$$

$$\rho_2\psi_{tt} - b\psi_{xx} + k(\varphi_x + \psi + lw) + \gamma\psi_t = 0 \quad \text{in } (0,\infty)\times(0,L) \tag{1.2}$$

$$\rho_1 w_{tt} - k_0(w_x - l\varphi)_x + kl(\varphi_x + \psi + lw) = 0 \quad \text{in } (0,\infty)\times(0,L) \tag{1.3}$$

where the functions w, φ, ψ are the longitudinal, vertical and shear angle displacements, respectively, and ρ_1, ρ_2, k, k_0, b, l and γ are positive constants related to physical properties of the beam. We consider the following initial conditions:

$$\varphi(0,\cdot) = \varphi_0, \qquad \varphi_t(0,\cdot) = \varphi_1, \qquad \psi(0,\cdot) = \psi_0, \qquad \psi_t(0,\cdot) = \psi_1, \qquad w(0,\cdot) = w_0, \qquad w_t(0,\cdot) = w_1$$

and the Dirichlet–Neumann–Neumann boundary conditions

$$\varphi(t,0) = \varphi(t,L) = \psi_x(t,0) = \psi_x(t,L) = w_x(t,0) = w_x(t,L) = 0 \quad \text{in } (0,\infty). \tag{1.4}$$

Concerning the asymptotic behavior of the Bresse system we have a few results. The most important is due to Liu and Rao [1]; the authors consider the thermoelastic Bresse system with two dissipative mechanisms. They showed that the exponential decay exists only when the velocities of the wave propagations are the same. For when the wave speeds are different they obtained a polynomial type of decay. This result was improved by Fatori and Muñoz Rivera [2]; the authors consider the thermoelastic Bresse system with one dissipative mechanism. For the case of the Bresse system given by (1.1)–(1.3) we can cite the work of Alabau Boussouira et al. [3]. There, the authors proved that, in general, the system is not exponentially stable but there exists polynomial decay with rates that depend on some particular relation between the coefficients. Using boundary conditions of Dirichlet–Dirichlet–Dirichlet type, they proved that the associated semigroup decays at a rate $t^{-1/6+\epsilon}$ or $t^{-1/3+\epsilon}$ for ϵ a small number.

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The main result of this work is to complete the analysis studying the optimal decay rate. More explicitly, we show, using the boundary conditions (1.4), that the decay of the solution is polynomial, with a rate that does not depend on the ϵ. That is, we prove that the associated semigroup decays at a rate $t^{-1/4}$ or $t^{-1/2}$ and for a particular relation between the coefficients, the decay rate obtained is optimal.

Our result on the polynomial stability is based on Theorem 2.4 in [4] and for the optimality of the polynomial decay rate we use the necessary condition given in [2, Theorem 5.3].

2. Preliminaries, and existence and uniqueness of solutions

The proof of the existence and uniqueness of solutions for the system (1.1)–(1.3) is obtained using semigroup techniques. The system (1.1)–(1.3) can be formulated as the following Cauchy problem:

$$U_t = \mathcal{A}U, \qquad U(0) = U_0$$

where $U = (\varphi, \varphi_t, \psi, \psi_t, w, w_t)'$, $U_0 = (\varphi_0, \varphi_1, \psi_0, \psi_1, w_0, w_1)'$, the prime is used to denote the transpose and $\mathcal{A} : D(\mathcal{A}) \subset \mathcal{H} \to \mathcal{H}$ is the (formal) differential operator

$$\mathcal{A} = \begin{pmatrix} 0 & I_d & 0 & 0 & 0 & 0 \\ \dfrac{k}{\rho_1}\partial_x^2 - \dfrac{k_0 l^2}{\rho_1} I_d & 0 & \dfrac{k}{\rho_1}\partial_x & 0 & \dfrac{(k+k_0)l}{\rho_1}\partial_x & 0 \\ 0 & 0 & 0 & I_d & 0 & 0 \\ -\dfrac{k}{\rho_2}\partial_x & 0 & \dfrac{b}{\rho_2}\partial_x^2 - \dfrac{k}{\rho_2} I_d & -\dfrac{\gamma}{\rho_2} I_d & -\dfrac{kl}{\rho_2} I_d & 0 \\ 0 & 0 & 0 & 0 & 0 & I_d \\ -\dfrac{(k_0+k)l}{\rho_1}\partial_x & 0 & -\dfrac{kl}{\rho_1} I_d & 0 & \dfrac{k_0}{\rho_1}\partial_x^2 - \dfrac{kl^2}{\rho_1} I_d & 0 \end{pmatrix}$$

where I_d is the identity operator. The domain of $\mathcal{A}$ is

$$D(\mathcal{A}) = \Big\{(\varphi, \tilde{\varphi}, \psi, \tilde{\psi}, w, \tilde{w})' \in \mathcal{H} : \varphi \in H^2(0,L) \cap H_0^1(0,L), \psi, w \in H^2(0,L), \\ \psi_x, w_x \in H_0^1(0,L), \tilde{\varphi} \in H_0^1(0,L), \tilde{\psi}, \tilde{w} \in H_*^1(0,L)\Big\}$$

where

$$\mathcal{H} = H_0^1(0,L) \times L^2(0,L) \times H_*^1(0,L) \times L_*^2(0,L) \times H_*^1(0,L) \times L_*^2(0,L)$$

is the Hilbert space with norm given by

$$\|U\|_{\mathcal{H}}^2 = \|(\varphi, \tilde{\varphi}, \psi, \tilde{\psi}, w, \tilde{w})'\|_{\mathcal{H}}^2 \\ = \rho_1\|\tilde{\varphi}\|_{L^2}^2 + \rho_2\|\tilde{\psi}\|_{L^2}^2 + \rho_1\|\tilde{w}\|_{L^2}^2 + b\|\psi_x\|_{L^2}^2 + k\|\varphi_x + \psi + lw\|_{L^2}^2 + k_0\|w_x - l\varphi\|_{L^2}^2$$

and $L_*^2(0,L) = \Big\{u \in L^2(0,L) : \int_0^L u(x)dx = 0\Big\}$

and $H_*^1(0,L) = L_*^2(0,L) \cap H^1(0,L)$.

It is not difficult to see that $\mathcal{A}$ is a dissipative operator in $\mathcal{H}$, that $0 \in \rho(\mathcal{A})$ and therefore, by the Lummer–Phillips Theorem (see [5], Theorem 4.3), the operator $\mathcal{A}$ generates a C_0-semigroup of contractions $S(t) = e^{\mathcal{A}t}$ on $\mathcal{H}$.

Thus, we have the following result.

Theorem 2.1. *Assume that* $(\varphi_0, \varphi_1, \psi_0, \psi_1, w_0, w_1) \in D(\mathcal{A})$*; then there exists a unique solution* $(\varphi, \varphi_t, \psi, \psi_t, w, w_t)$ *to the system (1.1)–(1.3) with boundary conditions (1.4), satisfying*

$$(\varphi, \varphi_t, \psi, \psi_t, w, w_t) \in C((0,\infty); D(\mathcal{A}) \cap C^1(0,\infty); \mathcal{H}).$$

3. Polynomial stability and the optimal decay rate

In [3] the authors proved that the semigroup associated with the system (1.1)–(1.3) with boundary conditions of the Dirichlet–Dirichlet–Dirichlet type or mixed boundary conditions (1.4) is polynomially stable provided

$$\frac{\rho_1}{\rho_2} = \frac{k}{b} \quad \text{and} \quad k = k_0 \tag{3.5}$$

and moreover they proved the lack of exponential stability when they considered the boundary condition (1.4).