Applied Mathematics Letters 25 (2012) 1166–1171

Contents lists available at SciVerse ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

The attracting set for impulsive stochastic difference equations with continuous time☆

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ARTICLE INFO

Article history:
Received 21 May 2011
Received in revised form 22 February 2012
Accepted 22 February 2012

Keywords:
Difference equation
Impulsive
Stochastic
Attracting set
Exponential stability

ABSTRACT

In this letter, an impulsive stochastic difference equation with continuous time is considered. By constructing an improved time-varying difference inequality, some sufficient criteria for the global attracting set and exponential stability in mean square are obtained. A numerical example is given to demonstrate the efficiency of the proposed methods.

1. Introduction

Difference equations with continuous time are difference equations in which the unknown function is a function of a continuous variable. These equations appear as natural descriptions of observed evolution phenomena in many branches of the natural sciences (see [1]). Difference equations with continuous time have attracted more and more attention from many researchers because of their plentiful dynamic behaviors and extensive applications.

Philos and Purnaras [2] obtained some results on the asymptotic behavior of scalar delay difference equations with continuous variables. Shaikhet (see [3,4]) derived a few of criteria for the stability of difference equations and stochastic difference equations with continuous variables by using different Lyapunov functionals. Romanenko [5] discussed the attractors of continuous difference equations. Rodkina [6] studied some asymptotic behaviors of solutions of stochastic difference equations. A lot of results on the oscillation of difference equations with continuous variables can be seen in [7–13]. Kolmanovskii et al. [14] discussed the mean square stability of difference equations with a stochastic delay. Yang and Xu [15] obtained some significant results on mean square exponential stability of impulsive stochastic difference equations by improving a difference inequality. Zhu [16] derived some criteria for invariant and attracting sets of impulsive delay difference equations with continuous variables. Bao et al. [17] studied the exponential stability in mean square of stochastic difference equations with continuous time.

However, to our knowledge, few studies have been done on the global attracting set of impulsive stochastic difference equations with continuous time. Motivated by this lack, our main aim in this letter is to present a new method for studying the global attracting set and exponential stability in mean square. After constructing an improved time-varying difference inequality, we derive several sufficient criteria for the global attracting set and exponential stability in mean square of

☆ This work was supported by the National Natural Science Foundation of China under Grants 10971147, 60974132, 10971240 and the Natural Science Foundation Project of CQ CSTC2011jjA00012.

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doi:10.1016/j.aml.2012.02.054

impulsive stochastic difference equations with continuous time. The techniques presented in this paper are still applicable to impulsive stochastic difference equations with discrete time. A numerical example is given to show the power of the proposed methods.

2. The model description and preliminaries

Throughout this letter, let R and R^+ be the sets of real numbers and nonnegative real numbers, respectively. R^n is the space of n-dimensional real column vectors. $PC[J, R] = \{\psi : J \to R | \psi(s)$ is continuous for all but at most countably many points $s \in J \subset R$ and at these $s \in J$, $\psi(s^+)$ and $\psi(s^-)$ exist and $\psi(s^+) = \psi(s)\}$, in which $\psi(s^+)$ and $\psi(s^-)$ denote the right-hand and left-hand limits of the function, respectively. Let $(\Omega, \mathcal{F}, \{\mathcal{F}\}_{t\geq 0}, \mathcal{P})$ be a complete probability space with a filtration $\{\mathcal{F}\}_{t\geq 0}$ satisfying right continuity and let $\mathcal{F}_0$ contain all $\mathcal{P}$-null sets. $PC_{\mathcal{F}_0}[[-h, 0], R]$ denotes the family of all bounded $\mathcal{F}_0$-measurable, $PC[[-h, 0], R]$-valued stochastic processes ϕ with the norm $\|\phi\| = \sup_{-h\leq s\leq 0} \mathbf{E}|\phi(s)|^2$, where $\mathbf{E}$ denotes the expectation of the stochastic process.

Consider a general impulsive stochastic difference equation with continuous time as follows:

$$\begin{cases} x(t+\sigma) = f(t, x, x(t-\tau_1), \ldots, x(t-\tau_m)) + g(t, x, x(t-\tau_1), \ldots, x(t-\tau_m))\xi(t+\sigma), & t \neq t_k, \\ x(t_k) = h_k(x(t_k^-)), & t = t_k, \\ x(s) = \phi(s), & -\tau - \sigma \leq s \leq 0, \end{cases} \tag{1}$$

where $f, g : R^+ \times R^{m+1} \to R$, $\tau = \max_{1\leq j\leq m} \tau_j$, and $\xi(t+\sigma)$ is a $\mathcal{F}_t$-measurable stationary and mutually independent stochastic process satisfying $\mathbf{E}\xi(t+\sigma) = 0$, $\mathbf{E}\xi^2(t+\sigma) = 1$. The fixed impulsive moments t_k satisfy $0 = t_0 < t_1 < t_2 < \cdots$ and $\lim_{k\to+\infty} t_k = +\infty$. Furthermore, we suppose that model (1) satisfies the following hypotheses.

H1. There exist some nonnegative functions $a_j(t)$, $b_j(t)$, $p(t)$ and $q(t)$ such that

$$|f(t, x(t), x(t-\tau_1), \ldots, x(t-\tau_m))| \leq \sum_{j=0}^{m} a_j(t)|x(t-\tau_j)| + p(t)$$

and

$$|g(t, x(t), x(t-\tau_1), \ldots, x(t-\tau_m))| \leq \sum_{j=0}^{m} b_j(t)|x(t-\tau_j)| + q(t).$$

Here $\tau_0 = 0$, $p(t)$ and $q(t)$ are bounded.

H2. There exist several constants $I_k \geq 1$ such that

$$|x(t_k)| = |h_k(x(t_k^-))| \leq I_k|x(t_k^-)|, \quad k = 1, 2, \ldots.$$

Some definitions are employed in this letter.

Definition 1. A set $D \subset PC_{\mathcal{F}_0}[[-\tau-\sigma, 0], R]$ is called a global attracting set of (1) if for any solution $x(t, \phi)$ with initial function $\phi \in PC_{\mathcal{F}_0}[[-\tau-\sigma, 0], R]$,

$$d(x(t, \phi), D) \longrightarrow 0, \quad \text{as } t \longrightarrow +\infty, \tag{2}$$

in which $d(x, D) = \inf_{y\in D} d(x, y)$, and $d(x, y)$ is the distance from x to y in $PC_{\mathcal{F}_0}[[-\tau-\sigma, 0], R]$.

Definition 2. The null solution of (1) is called global exponential stability in mean square if for any initial function $\phi \in PC_{\mathcal{F}_0}[[-\tau-\sigma, 0], R]$ there exists a pair of positive numbers K and γ such that

$$\mathbf{E}|x(t, \phi)|^2 \leq Ke^{-\gamma t}, \quad t \geq 0. \tag{3}$$

3. The main results

In order to study global attracting set, we need to make bounded estimations for all solutions of model (1). But it is very difficult to derive the estimations from previous results. Therefore, we introduce an improved time-varying difference inequality as follows.

Lemma 1. *Let $u(t)$ be a nonnegative function satisfying*

$$u(t+\sigma) \leq \sum_{j=0}^{m} a_j(t)u(t-h_j) + r(t), \quad t > 0, \tag{4}$$

in which $\sigma > 0$, $h_j \geq 0$ and:

(i) *$a_j(t) \in R^+$ for $j = 0, 1, \ldots, m$ and $\sup_{t\geq 0} \sum_{j=0}^{m} a_j(t) < 1$;*

(ii) *$r(t) \in R^+$ and $\sup_{t\geq 0} r(t) < +\infty$;*