Applied Mathematics Letters 24 (2011) 2153–2157

Contents lists available at ScienceDirect

Applied Mathematics Letters

journal homepage: www.elsevier.com/locate/aml

# The Hadamard product of meromorphic univalent functions defined by using convolution

R.M. El-Ashwah[a,*], M.K. Aouf[b]

[a] Math. Department, Fac. of Sci. (Damietta Branch), Mans. University, New Damietta 34517, Egypt

[b] Math. Department, Fac. of Sci., Mans. University, Mansoura 35516, Egypt

ARTICLE INFO

*Article history:*
Received 28 October 2009
Received in revised form 10 June 2011
Accepted 14 June 2011

*Keywords:*
Analytic
Meromorphic
Positive coefficients
Hadamard product

ABSTRACT

In this work the authors extend certain results concerning the Hadamard product for two classes related to starlike and convex univalent meromorphic functions with positive coefficients by using convolution.



## 1. Introduction

Throughout this work, let the functions of the form

$$\phi(z) = c_1 z - \sum_{n=2}^{\infty} c_n z^n \quad (c_1 > 0;\, c_n \geq 0), \tag{1.1}$$

and

$$\psi(z) = d_1 z - \sum_{n=2}^{\infty} d_n z^n \quad (d_1 > 0;\, d_n \geq 0) \tag{1.2}$$

be regular and univalent on the unit disc $U = \{z : |z| < 1\}$; also let

$$f(z) = \frac{a_0}{z} + \sum_{n=1}^{\infty} a_n z^n \quad (a_0 > 0;\, a_n \geq 0), \tag{1.3}$$

$$f_i(z) = \frac{a_{0,i}}{z} + \sum_{n=1}^{\infty} a_{n,i} z \quad (a_{0,i} > 0;\, a_{n,i} \geq 0), \tag{1.4}$$

$$g(z) = \frac{b_0}{z} + \sum_{n=1}^{\infty} b_n z^n \quad (b_0 > 0,\, b_n \geq 0), \tag{1.5}$$

* Corresponding author.
*E-mail addresses:* r_elashwah@yahoo.com, elashwah@mans.edu.eg (R.M. El-Ashwah), mkaouf127@yahoo.com (M.K. Aouf).

0893-9659/$ – see front matter 
doi:10.1016/j.aml.2011.06.017

and

$$g_j(z) = \frac{b_{0,j}}{z} + \sum_{n=1}^{\infty} b_{n,j} z^n \quad (b_{0,j} > 0; b_{n,j} \geq 0), \tag{1.6}$$

be regular and univalent in the punctured disc $U^* = \{z : 0 < |z| < 1\}$.

Denote by $\Sigma S_0^*(\alpha)$ the class of functions $f(z)$ which satisfies the condition

$$\mathrm{Re}\left\{-\frac{zf^{'}(z)}{f(z)}\right\} > \alpha \tag{1.7}$$

for some $\alpha$ $(0 \leq \alpha < 1)$ and for all $z \in U^*$.

Also let $\Sigma K_0(\alpha)$ be the class of functions $f(z)$ which satisfies the condition

$$\mathrm{Re}\left\{-\left(1 + \frac{zf^{''}(z)}{f'(z)}\right)\right\} > \alpha \tag{1.8}$$

for some $\alpha$ $(0 \leq \alpha < 1)$ and for all $z \in U^*$.

The functions $\Sigma S_0^*(\alpha)$ and $\Sigma K_0(\alpha)$ are, respectively, called meromorphically starlike and meromorphically convex of order $\alpha$ with positive coefficients in $U^*$.

The quasi-Hadamard product of two or more functions has recently been defined and used by Owa [1], Kumar [2–4], Aouf and Darwish [5], Darwish [6], Hossen [7] and Sekine [8]. Accordingly, the quasi-Hadamard product of two functions $\phi(z)$ and $\psi(z)$ given by (1.1) and (1.2) is defined by

$$\phi * \psi(z) = c_1 d_1 z - \sum_{n=2}^{\infty} c_n d_n z^n. \tag{1.9}$$

Let us define the Hadamard product of two meromorphic univalent functions $f(z)$ and $g(z)$ by

$$f * g(z) = \frac{a_0 b_0}{z} + \sum_{n=1}^{\infty} a_n b_n z^n. \tag{1.10}$$

The Hadamard product of more than two meromorphic functions can be defined similarly.

Let $\varphi(z)$ be a fixed function of the form

$$\varphi(z) = \frac{a_0}{z} + \sum_{m=1}^{\infty} c_n z^n \quad (a_0 > 0; c_n \geq c_1 > 0; n \geq 1). \tag{1.11}$$

Using the function defined by (1.11), we now define the following new classes.

**Definition 1.** We have that a function $f(z) \in \sum_{\varphi} S^*(c_n, \delta)$ $(c_n \geq c_1 > 0; n \geq 1)$ if and only if

$$\sum_{m=1}^{\infty} c_n a_n \leq \delta a_0 \quad (\delta > 0). \tag{1.12}$$

**Definition 2.** We have that a function $f(z) \in \sum_{\varphi} C(c_n, \delta)$ $(c_n \geq c_1 > 0; n \geq 1)$ if and only if

$$\sum_{m=1}^{\infty} n c_n a_n \leq \delta a_0 \quad (\delta > 0). \tag{1.13}$$

**Definition 3.** We have that a function $f(z) \in \sum_{\varphi}^{k}(c_n, \delta)$ $(c_n \geq c_1 > 0; n \geq 1)$ if and only if

$$\sum_{m=1}^{\infty} n^k c_n a_n \leq \delta a_0 \quad (\delta > 0), \tag{1.14}$$

where $k$ is any fixed nonnegative real number.

For suitable choices of $c_n, \delta$ and $k$ we obtain various classes of meromorphic univalent functions studied by various authors as follows: