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New results on almost periodic solutions for a class of nonlinear Duffing equations with a deviating argument[☆]

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Abstract

In this work a class of nonlinear Duffing equations with a deviating argument are considered. Some sufficient conditions for the existence of almost periodic solutions are established, which are new and complement previously known results.

Keywords: Nonlinear Duffing equations; Almost periodic solution; Existence; Deviating argument

1. Introduction

Consider the following model for a nonlinear Duffing equation with a deviating argument:

$$x''(t) - ax(t) + bx^m(t - \tau(t)) = p(t), \tag{1.1}$$

where $\tau(t)$ and $p(t)$ are almost periodic functions on R, $m > 1$ is an integer, $a > 0$ and $b \neq 0$ are constants.

In recent years, the dynamic behaviors of nonlinear Duffing equations have been widely investigated in [1–4] due to the application in many fields such as physics, mechanics and the engineering technique fields. In such applications, it is important to know of the existence of almost periodic solutions for nonlinear Duffing equations and some results on the existence of the almost periodic solutions were obtained in the literature. We refer the reader to [5–7] and the references cited therein. Suppose that the following condition:
(H_0) $a = b = 1,\ \tau$ is a constant, and

$$\sup_{t\in R} |p(t)| \leq \left(\frac{1}{m}\right)^{\frac{1}{m-1}} \left(1 - \frac{1}{m}\right) \tag{1.2}$$

is satisfied. The authors of [6,7] obtained some sufficient conditions ensuring the existence of almost periodic solutions for Eq. (1.1). However, to the best of our knowledge, few authors have considered the problem of almost periodic

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solutions for Eq. (1.1) without the assumption (H_0). Thus, it is worthwhile to continue to investigate the existence of almost periodic solutions of Eq. (1.1) in this case.

The main purpose of this work is to give new criteria for the existence of almost periodic solutions for Eq. (1.1). By applying some new mathematical analysis techniques, without assuming (H_0), we derive some sufficient conditions ensuring the existence of almost periodic solutions for Eq. (1.1), which are new and complement previously known results. Moreover, an example is also provided to illustrate the effectiveness of our results.

Define

$$y = \frac{dx}{dt} + \delta x, \tag{1.3}$$

where $\delta > 1$ is a constant; we can transform (1.1) into the following system:

$$\begin{cases} \dfrac{dx(t)}{dt} = -\delta x(t) + y(t), \\ \dfrac{dy(t)}{dt} = \delta y(t) + (a - \delta^2)x(t) - bx^m(t - \tau(t)) + p(t). \end{cases} \tag{1.4}$$

For convenience, we introduce some notation. We will use $X = (x_1, \ x_2)^T \in R^2$ to denote a column vector, in which the symbol (^{T}) denotes the transpose of a vector. We let $|X|$ denote the absolute-value vector given by $|X| = (|x_1|, \ |x_2|)^T$, and define $\|X\| = \max_{1\le i\le 2} |x_i|$. A vector $X \ge 0$ means that all entries of x are greater than or equal to zero. $X > 0$ is defined similarly. For vectors X and Y, $X \ge Y$ (resp. $X > Y$) means that $X - Y \ge 0$ (resp. $X - Y > 0$).

Throughout this work, we set

$$B = \{\varphi | \varphi = (\varphi_1(t), \varphi_2(t))^T\},$$

where φ is an almost periodic function on R. For $\forall \varphi \in B$, we define the induced modulus $\|\varphi\|_B = \sup_{t\in R} \|\varphi(t)\|$; then B is a Banach space.

Definition 1 (*See [8,9]*). Let $u(t) : R \longrightarrow R^n$ be continuous in t. $u(t)$ is said to be almost periodic on R if, for any $\varepsilon > 0$, the set $T(u, \varepsilon) = \{\delta : \|u(t + \delta) - u(t)\| < \varepsilon, \forall t \in R\}$ is relatively dense, i.e., for $\forall \varepsilon > 0$, it is possible to find a real number $l = l(\varepsilon) > 0$ where, for any interval with length $l(\varepsilon)$, there exists a number $\delta = \delta(\varepsilon)$ in this interval such that $\|u(t + \delta) - u(t)\| < \varepsilon, \forall t \in R$.

Definition 2 (*See [8,9]*). Let $x \in R^n$ and $Q(t)$ be an $n \times n$ continuous matrix defined on R. The linear system

$$x'(t) = Q(t)x(t) \tag{1.5}$$

is said to admit an exponential dichotomy on R if there exist positive constants k, α, projection P and the fundamental solution matrix $X(t)$ of (1.5) satisfying

$$\|X(t)PX^{-1}(s)\| \le ke^{-\alpha(t-s)} \quad \text{for } t \ge s,$$
$$\|X(t)(I - P)X^{-1}(s)\| \le ke^{-\alpha(s-t)} \quad \text{for } t \le s.$$

Lemma 1.1 (*See [8,9]*). *If the linear system (1.5) admits an exponential dichotomy, then the almost periodic system*

$$x'(t) = Q(t)x + g(t) \tag{1.6}$$

has a unique almost periodic solution $x(t)$, and

$$x(t) = \int_{-\infty}^{t} X(t)PX^{-1}(s)g(s)ds - \int_{t}^{+\infty} X(t)(I - P)X^{-1}(s)g(s)ds. \tag{1.7}$$

Lemma 1.2 (*See [8,9]*). *Let $Q(t) = (q_{ij})$ be an $n \times n$ almost periodic matrix defined on R, and let there exist a positive constant ν such that*

$$|q_{ii}(t)| - \sum_{j=1, j\ne i}^{n} |q_{ij}(t)| \ge \nu, \quad i = 1, 2, \ldots, n.$$