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Period-doubling bifurcation of a discrete metapopulation model with a delay in the dispersion terms[☆]

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Abstract

In this work the period-doubling bifurcation of a discrete metapopulation with delay in the dispersion terms is discussed. By using the central manifold method, the period-doubling bifurcation can be analyzed from the viewpoint of the dynamical system. Intensive simulation on this model shows the dynamics of the metapopulation is similar to that of a single logistic model as the bifurcation parameter μ increases when $0 \le b < 1/2$, where b is the dispersion parameter.

Keywords: Metapopulation; Logistic; Dispersion; Period-doubling bifurcation; Fixed point; Periodic orbits; Stability

1. Introduction

Metapopulation is an important concept in several ecological fields, including population ecology, landscape ecology, and conservation biology, which provides a theoretical framework for studying spatially structured populations. There have been many studies on metapopulations using continuous time models; see [5–8]. But in the context of the discrete models, there are relatively few contributions in the literature. Recently, Gyllenberg et al. [3] considered a two-patch discrete time metapopulation model of coupled logistic difference equations and gave a characterization of the fixed point and 2-periodic orbits. Yakubu and Castillo-Chavez [9] studied a more general metapopulation model over N patches. The effects of synchronous dispersal on discrete time metapopulation dynamics with local (patch) dynamics of the same (compensatory or overcompensatory) or mixed (compensatory and overcompensatory) types are explored in [9]. More recently, Huang and Zou [4] proposed the following model system:

$$\begin{cases} x(n+1) = \mu x(n)(1-x(n)) + d_2 y(n-k_2) - d_1 x(n-k_1), \\ y(n+1) = \nu y(n)(1-y(n)) + d_1 x(n-k_1) - d_2 y(n-k_2) \end{cases} \tag{1}$$

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where $d_1, d_2 \geq 0$ represent the dispersion rate, $0 < \mu, \nu < 4$ represent the growth rate, and $0 \leq x(n), y(n) < 1$ represent the population density of each subpopulation after n generations. The model carries a delay in the dispersion terms to account for long distance dispersion. Only a special case of (1): $k_1 = k_2 = 1$ and $d_1 = d_2 = b$ (meaning symmetric dispersal) is considered, and the impact of the dispersion on the global dynamics of the metapopulation is obtained in [4]. It is very hard and challenging work to study system (1) directly either in theory or in simulation. In order to avoid the important biological features from being hidden behind the complexity caused by high dimensions and multiparameter, Zeng et al. in [10] just discuss Hopf bifurcation of a special case of (1): $k_1 = k_2 = 1$, $\nu = \mu$ and $d_1 = d_2 = b$, that is, the following model system:

$$\begin{cases} x(n+1) = \mu x(n)(1 - x(n)) + b[y(n-1) - x(n-1)], \\ y(n+1) = \mu y(n)(1 - y(n)) + b[x(n-1) - y(n-1)]. \end{cases} \tag{2}$$

In this work, we will deal with the period-doubling bifurcation of this model. Note that when $b = 0$, there is no coupling and each subpopulation in (2) is governed by a well known discrete logistic equation of the form

$$u(n+1) = \mu u(n)(1 - u(n)). \tag{3}$$

This one-dimensional dynamical system has been studied extensively and its dynamics, such as the period-doubling process from a stable 2^{n-1}-periodic orbit to a stable 2^n-periodic orbit and a route to chaos as the parameter μ increases in [1], is well understood. Comparing our results for (2) with that for (3), we find that when $0 < b < 1/2$ the dynamics of (2) is similar to that of (3), but the period-doubling bifurcation cascade for (2) occurs earlier than that for the system (3). We can analytically prove that there exist period-doubling bifurcations at the positive fixed point and 2-periodic orbit of (2) when $0 \leq b < 1/2$. By simulation, we also make a conjecture that the system (2) undergoes a cascade of period-doubling bifurcation and finally becomes chaotic as μ increases if $0 \leq b < 1/2$. From the ecology viewpoint, when μ is increased, each subpopulation will oscillate in cycles of period 2^n (where n increases from 1 to infinity), and finally vary randomly and boundedly.

The rest of this work is organized as follows. Section 2 reviews some known results on the model (2). Section 3 is devoted to our main results. By using a change of coordinates and the central manifold method, the first period-doubling bifurcation and the second period-doubling bifurcation of (2) are analyzed in Section 3. Finally, some discussions and conjectures are given in Section 4.

2. Preliminaries

In this section, we first review some results about the model (2); for details, see [4,10]. We only consider the nonnegative solutions of (2) from the viewpoint of ecology, i.e. $x(n), y(n) \geq 0$ for any integer n. When $b = 0$, the dynamics of (2) is determined by the one-dimensional logistic equation (3). It is well known that the logistic equation undergoes a period-doubling cascade as μ increases, that is, there exists a sequence $\mu_0 = 1 < \mu_1 = 3 < \mu_2 = 1 + \sqrt{6} < \mu_3 < \cdots < \mu_n < \cdots < \mu_\infty \approx 3.56994$ such that when $\mu \in (\mu_n, \mu_{n+1})$, $n = 0, 1, 2, \ldots$, (3) has a unique stable 2^n-periodic orbit.

For $\mu \in (\mu_n, \mu_{n+1})$, let $\{u_i, i = 1, 2, \ldots, 2^n\}$ be the corresponding stable 2^n-periodic orbit of the logistic equation (3). Then $\{(u_i, u_i), i = 1, 2, \ldots, 2^n\}$ is a 2^n-periodic orbit of (2) for any $b > 0$. Letting

$$w_1(n) = \frac{x(n) + y(n)}{2}, \qquad w_2(n) = \frac{x(n-1) - y(n-1)}{2}, \qquad w_3(n) = \frac{x(n) - y(n)}{2},$$

we can rewrite the difference system (2) as the three-dimensional discrete dynamical system

$$\begin{pmatrix} w_1(n+1) \\ w_2(n+1) \\ w_3(n+1) \end{pmatrix} = \begin{pmatrix} \mu(w_1(n) - w_1(n)^2 - w_3(n)^2) \\ w_3(n) \\ \mu(w_3(n) - 2w_1(n)w_3(n)) - 2bw_2(n) \end{pmatrix} \triangleq G \begin{pmatrix} w_1(n) \\ w_2(n) \\ w_3(n) \end{pmatrix}. \tag{4}$$

Since the dynamics of the system (2) is qualitatively the same as that of the system (4), we only need to analyze the system (4) qualitatively. By the above transformation, the 2^n-periodic orbit $\{(u_i, u_i), i = 1, 2, \ldots, 2^n\}$ of the system (2) is transformed to the 2^n-periodic orbit $\{W_{2^n}^i, i = 1, 2, \ldots, 2^n\}$ of the system (4), where $W_{2^n}^i = (u_i, 0, 0)^T$. By a simple computation, one can obtain the positive fixed point $W_1^1 = (1 - \frac{1}{\mu}, 0, 0)^T$, $\mu > \mu_0$ and the 2-periodic orbit $\{W_2^i = (u_i, 0, 0)^T, i = 1, 2\}$, $\mu > \mu_1$, where