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Simple transformation functions for finding better minima[☆]

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Abstract

This work presents two transformation functions, the α-function and the M-function, for finding better minimizers in global optimization. We prove that under some general assumptions these functions possess the characters of both tunnelling functions and filled functions. Numerical tests from some test functions show that our transformation functions are very effective in finding better minima.

Keywords: Mathematical programming; Global optimization; Filled function; Tunnelling function

1. Introduction

Because of advances in science, economics and engineering, studies on global optimization for the multi-minimum nonlinear programming problem

$$\min\{f(x) : x \in R^n\}$$

have become a topic of great concern. Since the 1970's, many new theoretical and computational contributions such as Dixon and Szegö [1]; Horst, Pardalos and Thoai [6] and so on have been developed. In particular, the tunnelling function proposed by Levy and Montalvo [7] and the filled function introduced by Ge and Qin [2] are two practical and useful tools for global optimization.

The tunnelling algorithm is composed of two phases, a minimizing phase and a tunnelling phase. These two phases are used alternately to search for the better minimizer. Suppose minimizers $x_1^*, \ldots, x_{m-1}^*$ have been found. In the first phase, a classical algorithm such as Newton's method or the steepest descent method can be used to find next local minimizer x_m^* of the objective function $f(x)$. In the second phase, the search for roots of a defined auxiliary function, the tunnelling function $T(x)$, is carried out. The tunnelling function in Levy and Montalvo [7] and Yao [10] is defined

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as

$$T(x) = \frac{f(x) - f(x_m^*)}{[(x - x_1^*)^T (x - x_1^*)]^{\alpha_1} \cdots [(x - x_m^*)^T (x - x_m^*)]^{\alpha_m}}. \tag{1}$$

If we can get $\bar{x}$ such that $\bar{x} \notin \{x_1^*, \ldots, x_m^*\}$ and $T(\bar{x}) \leq 0$, i.e., $f(\bar{x}) \leq f(x_m^*)$, then $\bar{x}$ is a new starting point for the next iteration. The denominator of (1) is a pole at x_i^* $(i = 1, \ldots, m)$ with power α_i which prevents x_i^* from being a zero of the tunnelling function.

The filled function method is essentially similar to that for the tunnelling function. The only difference is the filled function that is used for finding $\bar{x}$ in phase II. There are various filled functions according to their expressions. The following filled function was applied by Han and Han [5]:

$$P(x, r, \rho) = \frac{1}{r + f(x)} \exp\left(-\frac{\|x - x^*\|^2}{\rho}\right), \tag{2}$$

and another filled function with prefixed point x_0

$$U(x, A, h) = \eta(\|x - x_0\|)\varphi(A[f(x) - f(x^*) + h]) \tag{3}$$

was proposed by Ge and Qin [4], Zhang, Ng, Li and Tian [11] and further discussed by Lucidi and Piccialli [9].

No matter what the expressions for the filled functions are (such as (2) and (3)), they possess the following common properties:

(i) x^* is a maximizer of the filled functions,
(ii) the filled function has no stationary point in $\{x : f(x) \geq f(x^*), x \neq x^*\}$, and
(iii) if $f(x^*)$ is not a global minimum, the filled function has either a minimizer in $\{x : f(x) < f(x^*)\}$ or a stationary point along the ray $x^* \to x'$, where x' is in a lower basin.

In general, there are two difficulties in global optimization. The first is how to leave the local minimizer to go to a better one. The second is how to check whether the current minimizer is a global solution of the problem. We pay our main attention to the first issue in this work.

This work is organized as follows. In Section 2, we introduce our problem and some assumptions. In Sections 3 and 4, we present the α-function and the M-function, and discuss their properties. In Section 5, we state our algorithm based on the α-function or M-function and make a numerical test. Last, in Section 6, we give our conclusion.

2. Problem and assumptions

Consider the unconstrained optimal problem

$$\min\{f(x) : x \in \Omega\} \tag{4}$$

where $f : R^n \to R$, and $\Omega \subset R^n$ is a large enough bounded closed region. We need the following assumptions:

Assumption 1. $f(x)$ is continuously differentiable in R^n and there exists a $K > 0$ such that $\|\nabla f(x)\| \leq K, \forall x \in R^n$.

Assumption 2. $f(x) \to +\infty$ as $\|x\| \to +\infty$, namely, $f(x)$ is a coercive function.

Assumption 3. $f(x)$ has a finite number of different minimal function values.

Assumption 3 is weaker compared with the assumptions used in many papers, such as [3,7] which assumes that $f(x)$ has a finite number of minimizers. Assumption 2 implies that there exists a bounded closed domain $\Omega \subset R^n$ that contains all minimizers of $f(x)$ in R^n. We only need to consider a bounded closed domain Ω because all minimizers of problem (4) are the inner points of Ω. So, the problem (4) is equivalent to

$$\min\{f(x) : x \in R^n\}.$$

Suppose that x_1^* is a local minimizer but not a global one. If there is an auxiliary function based on x_1^* that can be used to force the sequence of iterative points to leave the basin containing x_1^* to go to another point x' which is