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Convergence rate analysis of discrete-time Markovian jump systems☆

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ABSTRACT

This paper is concerned with convergence rate analysis of a discrete-time Markovian jump linear system. First, two eigenvalue sets of some operator associated with the Markovian jump linear system are defined to characterize its stability. It is shown that the fastest and slowest convergence rate of the Markovian jump system can be determined by the eigenvalues having minimal modulus and maximal modulus, respectively. Finally, a linear matrix inequality based approach is established to design controllers such that the closed-loop system has a guaranteed convergence rate. A numerical example is carried out to illustrate the effectiveness of the proposed approach.

1. Introduction

Stochastic systems have attracted increasing attention in recent years (*e.g.*, [1–3]). Especially, as one of the most important types of stochastic system, Markovian jump systems have received considerable research recently (see, for example, [4–6] and the references therein). The interest of this topic lies in several aspects and one of them is that plenty of practical control systems can be modeled by Markovian jump systems with Markovian jumping parameters. These practical systems include network control systems, a model with random abrupt changes, sudden environmental changes, abrupt variations of the operating point and so on. Due to its wide applications, many control problems that have been extensively studied for normal systems are extended to this kind of systems. These problems include stability analysis and stabilization [7,8], filter design [9], robust control [10,11] and so on [12–14].

Similar to deterministic systems, the stability problem is also one of the most important issues required to be studied. In fact, problem of stability analysis and stabilization of stochastic systems with Markovian jumping parameters has been deeply studied by many researchers. For instance, by applying the Lyapunov stability approach, necessary and sufficient conditions for mean square stability were given in [7]. Ref. [15] gives necessary and sufficient conditions for the mean square stability of discrete-time Markovian jump linear systems based on the maximal real part of an augmented matrix. Besides, [5] addresses the problem of robust stability and stabilization of discrete-time Markovian jump linear systems with uncertain parameters by using a linear matrix inequalities (LMIs) based method. We notice that a similar situation exits in general stochastic systems. For example, the well-known spectral characterization of deterministic systems was generalized to stochastic systems in several aspects (see [16] and the references therein).

In this note, we are also interested in the stability of Markovian jump linear systems and our aim is to provide a convergence rate analysis of a discrete-time Markovian jump linear system in the mean square sense. To this end, we define two classes of eigenvalue sets for some positive operator associated with the Markovian jump system. By exploring

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the properties of these eigenvalue sets, the fastest and slowest convergence rates of the discrete-time Markovian jump linear system are characterized by respectively the minimal modulus and maximal modulus of the eigenvalues within some eigenvalue set. Finally, an LMIs-based approach is established to design controllers such that the closed-loop system has guaranteed convergence rate. A numerical example is provided to illustrate the effectiveness of the proposed approach.

Notation. We let $\mathbf{R}^{n\times m}$ and $\mathbf{C}^{n\times m}$ represent respectively the sets consisting of $n \times m$ real and complex matrices. For any matrix A, we use $\operatorname{rank}(A)$, $\operatorname{tr}(A)$ and A^{T} to denote its rank, trace and transpose, respectively. Also, $\mathbf{C}^{\odot} = \{\lambda : |\lambda| < 1\}$ and $\otimes$ denotes the Kronecker product. The set $\mathbf{S}^{n\times n}$ represents the set of $n \times n$ symmetric matrices. Associated with $\mathbf{S}^{n\times n}$, we define $\mathbf{S}_{+}^{n\times n} = \left\{A : A \in \mathbf{S}^{n\times n},\ A \geq 0\right\}$. We let $\mathcal{C}^{n\times m}$ and $\mathcal{S}^{n\times n}$ denote respectively the linear space made up of all sequence of complex and symmetric matrices $C = (C_1, C_2, \ldots, C_N)$ with $C_i \in \mathbf{C}^{n\times m}$ and $C_i \in \mathbf{S}^{n\times n}$, where $N \geq 1$ is an integer. Similarly, $\mathcal{S}_{+}^{n\times n}$ can be defined in the same way. For any matrix $X = [x_1\ x_2\ \cdots\ x_n] \in \mathbf{C}^{n\times n}$, the stretching function is defined as

$$\operatorname{vec}(\cdot) = \left[x_1^{\mathrm{T}}\ x_2^{\mathrm{T}}\ \cdots\ x_n^{\mathrm{T}}\right]^{\mathrm{T}} : \mathbf{C}^{n\times n} \to \mathbf{C}^{n^2}.$$

Let $X = \left[x_{ij}\right] \in \mathbf{S}^{n\times n}$. Then the symmetric stretching function is defined as

$$\operatorname{svec}(\cdot) = [x_{11}\ x_{12}\ x_{22}\ \cdots\ x_{1n}\ x_{2n}\ \cdots\ x_{nn}]^{\mathrm{T}} : \mathbf{S}^{n\times n} \to \mathbf{C}^{\frac{n\times(n+1)}{2}}.$$

For any $X = (X_1, X_2, \ldots, X_N) \in \mathcal{C}^{n\times n}$ and $X \in \mathcal{S}^{n\times n}$, we denote respectively the operators $\varphi(\cdot) : \mathcal{C}^{n\times n} \to \mathbf{C}^{n^2 N}$ and $\varphi_s(\cdot) : \mathcal{S}^{n\times n} \to \mathbf{C}^{\frac{n\times(n+1)}{2}N}$ as

$$\varphi(X) = \left[\operatorname{vec}^{\mathrm{T}}(X_1) \quad \operatorname{vec}^{\mathrm{T}}(X_2) \quad \cdots \quad \operatorname{vec}^{\mathrm{T}}(X_N)\right]^{\mathrm{T}},$$
$$\varphi_s(X) = \left[\operatorname{svec}^{\mathrm{T}}(X_1) \quad \operatorname{svec}^{\mathrm{T}}(X_2) \quad \cdots \quad \operatorname{svec}^{\mathrm{T}}(X_N)\right]^{\mathrm{T}}.$$

Finally, for any $X \in \mathbf{S}_{+}^{n\times n}$, the norm of X is defined as $\|X\| = \operatorname{tr}(X)$. Consequently, for any $Q = (Q_1, Q_2, \ldots, Q_N) \in \mathcal{S}_{+}^{n\times n}$, the norm of Q is defined as

$$\|Q\| \triangleq \sum_{i=1}^{N} \|Q_i\| = \sum_{i=1}^{N} \operatorname{tr}(Q_i) = \operatorname{tr}\left(\sum_{i=1}^{N} Q_i\right) = \left\|\sum_{i=1}^{N} Q_i\right\|. \tag{1}$$

2. Stability properties of the DMJS

Let $\{\theta(k),\ k \geq 0\}$ be a discrete-time homogeneous Markov chain with finite state space $\mathcal{N} = \{1, 2, \ldots, N\}$ and stationary transition probability matrix $\Pi = \left[\pi_{ij}\right],\ i, j \in \mathcal{N}$ where $\Pr\{\theta(k+1) = j\,|\theta(k) = i\} = \pi_{ij} \geq 0,\ \forall i, j \in \mathcal{N}$. The discrete-time Markovian jump system (DMJS) on a probability space $(\Omega, \mathcal{E}, \mathcal{P})$ where Ω is the sample space, $\mathcal{E}$ is the algebra of events and $\mathcal{P}$ is the probability measure defined on $\mathcal{E}$, respectively, is given by

$$x(k+1) = A_{\theta(k)}x(k) + B_{\theta(k)}u(k), \qquad \theta(0) = \theta_0, \qquad x(0) = x_0, \tag{2}$$

where $x(k) \in \mathbf{R}^n$ is the state, $u(k) \in \mathbf{R}^m$ is the control input. For each possible value of $\theta(k) = i,\ i \in \mathcal{N}$, we denote the matrices associated with the "i-th mode" by $A_i = A_{\theta(k)=i}$ and $B_i = B_{\theta(k)=i}$, $i \in \mathcal{N}$, where $A_i \in \mathbf{R}^{n\times n}$ and $B_i \in \mathbf{R}^{n\times m},\ i \in \mathcal{N}$ are constant matrices.

We first introduce the stability concept for the DMJS (2).

Definition 1. The DMJS (2) with $u(k) = 0$ is asymptotically mean square stable (AMSS) if for any $x_0 \in \mathbf{R}^n$ and $\theta_0 \in \mathcal{N}$, there holds

$$\lim_{k\to\infty} \mathbf{E}\left\{\|x(k)\|^2\right\} = 0,$$

where $x(k) = x(k, x_0, \theta_0)$ is a sample solution of the DMJS (2).

Let

$$\mathscr{L} = (\mathscr{L}_1(X), \mathscr{L}_2(X), \ldots, \mathscr{L}_N(X))$$

with

$$\mathscr{L}_j(X) = \sum_{i=1}^{N} \pi_{ij} A_i X_i A_i^{\mathrm{T}}, \quad j \in \mathcal{N}.$$

Associated with the operator $\mathscr{L}$, we define the eigenvalue set in $\mathcal{C}^{n\times n}$ as

$$\sigma_{\mathcal{C}}(\mathscr{L}) = \left\{\lambda : \mathscr{L}(X) = \lambda X,\ 0 \neq X \in \mathcal{C}^{n\times n}\right\}. \tag{3}$$

For each λ in $\sigma_{\mathcal{C}}(\mathscr{L})$, the associated nonzero matrix X is called an eigenvector of $\mathscr{L}$ corresponding to λ. By definition (3) and using the operator $\varphi(\cdot)$, we have