Available online at www.sciencedirect.com
ScienceDirect
Applied Mathematics Letters 21 (2008) 814–819
Applied Mathematics Letters
www.elsevier.com/locate/aml

A note on convexity and semicontinuity of fuzzy mappings[☆,☆☆]

Yu-Ru Syau[a,*], E. Stanley Lee[b]

[a] Department of Information Management, National Formosa University, Huwei, Yunlin 63201, Taiwan
[b] Department of Industrial and Manufacturing Systems Engineering, Kansas State University, Manhattan, KS 66506, United States

Received 13 September 2007; accepted 13 September 2007

Abstract

By using parameterized representation of fuzzy numbers, criteria for a lower semicontinuous fuzzy mapping defined on a non-empty convex subset of R^n to be a convex fuzzy mapping are obtained.

Keywords: Convexity; Fuzzy mapping; Fuzzy numbers; Convex fuzzy mapping; Semicontinuity

1. Introduction

Convexity and semicontinuity of fuzzy mappings play central roles in fuzzy mathematics and fuzzy optimization. The concept of convex fuzzy mappings defined through the “fuzz-max” order on fuzzy numbers were studied by several authors, including Furukawa [1], Nanda [2], Syau [3,4], and Wang and Wu [5], aiming at applications to fuzzy nonlinear programming. The concept of upper and lower semicontinuity of fuzzy mappings based on the Hausdorff separation was introduced by Diamond and Kloeden [6]. Recently, Bao and Wu [7] introduced a new concept of upper and lower semicontinuity of fuzzy mappings through the “fuzz-max” order on fuzzy numbers, and obtained the criteria for convex fuzzy mappings under upper and lower semicontinuity conditions, respectively. In an earlier paper [8], we redefined the upper and lower semicontinuity of fuzzy mappings of Bao and Wu [7] by using the concept of parameterized triples of fuzzy numbers.

Bao and Wu [7] established the criteria for a lower semicontinuous fuzzy mapping defined on a non-empty closed convex subset, say C, of R^n to be a convex fuzzy mapping. In this paper, by using parameterized representation of fuzzy numbers, we give the criteria for a lower semicontinuous fuzzy mapping defined on a non-empty convex subset of R^n to be a convex fuzzy mapping. In other words, we deleted the requirement of the closed condition on C. That is, the set C only needs to be a non-empty convex subset of R^n.

☆ Supported by the National Science Council of Taiwan under contract NSC 96-2221-E-150-012.
☆☆ This work was carried out while the first author was visiting the Department of Industrial and Manufacturing Systems Engineering, Kansas State University.
* Corresponding author.
E-mail address: yrsyau@nfu.edu.tw (Y.-R. Syau).

0893-9659/$ - see front matter
doi:10.1016/j.aml.2007.09.003

2. Preliminaries

In this section, for convenience, several definitions and results without proof from [3,7–12] are summarized below.

Let R^n denote the n-dimensional Euclidean space. In what follows, let S be a non-empty subset of R^n. For any $x \in R^n$ and $\delta > 0$, let

$$B_\delta(x) = \{y \in R^n : \|y - x\| < \delta\},$$

where $\|\cdot\|$ being the Euclidean norm on R^n.

First, we recall the definitions of upper and lower semicontinuous real-valued functions.

Definition 2.1. A real-valued function $f : S \to R^1$ is said to be

(1) upper semicontinuous at $x_0 \in S$ if $\forall \varepsilon > 0$, there exists a $\delta = \delta(x_0, \varepsilon) > 0$ such that

$$f(x) < f(x_0) + \varepsilon \quad \text{whenever } x \in S \cap B_\delta(x_0).$$

f is upper semicontinuous on S if it is upper semicontinuous at each point of S.

(2) lower semicontinuous at $x_0 \in S$ if $\forall\, \varepsilon > 0$, there exists a $\delta = \delta(x_0, \varepsilon) > 0$ such that

$$f(x) > f(x_0) - \varepsilon \quad \text{whenever } x \in S \cap B_\delta(x_0).$$

f is lower semicontinuous on S if it is lower semicontinuous at each point of S.

The support, $\operatorname{supp}(\mu)$, of a fuzzy set $\mu : R^n \to I = [0, 1]$ is defined as

$$\operatorname{supp}(\mu) = \{x \in R^n : \mu(x) > 0\}.$$

A fuzzy set $\mu : R^n \to I$ is normal if $[\mu]_1 \neq \emptyset$. A fuzzy number we treat in this study is a fuzzy set $\mu : R^1 \to I$ which is normal, has bounded support, and is upper semicontinuous and quasiconcave as a function on its support.

Let $\alpha \in I$. The α-level set of a fuzzy set $\mu : R^n \to I$, denoted by $[\mu]_\alpha$, is defined as

$$[\mu]_\alpha = \begin{cases} \{x \in R^n : \mu(x) \geq \alpha\}, & \text{if } 0 < \alpha \leq 1; \\ \operatorname{cl}(\operatorname{supp}(\mu)), & \text{if } \alpha = 0, \end{cases}$$

where $\operatorname{cl}(\operatorname{supp}(\mu))$ denotes the closure of $\operatorname{supp}(\mu)$.

Denote by $\mathcal{F}$ the set of all fuzzy numbers. In this paper, we consider mappings F from a non-empty subset of R^n into $\mathcal{F}$. We call such a mapping a fuzzy mapping. It is clear that each $r \in R^1$ can be considered as a fuzzy number $\tilde{r}$ defined by

$$\tilde{r}(t) = \begin{cases} 1, & \text{if } t = r; \\ 0, & \text{if } t \neq r, \end{cases}$$

hence, each real-valued function can be considered as a fuzzy mapping.

It can be easily verified [9] that a fuzzy set $\mu : R^1 \to I$ is a fuzzy number if and only if (i) $[\mu]_\alpha$ is a closed and bounded interval for each $\alpha \in I$, and (ii) $[\mu]_1 \neq \emptyset$. Thus we can identify a fuzzy number μ with the parameterized triples

$$\{(a(\alpha), b(\alpha), \alpha) : \alpha \in I\},$$

where $a(\alpha)$ and $b(\alpha)$ denote the left- and right-hand endpoints of $[\mu]_\alpha$, respectively, for each $\alpha \in I$.

Definition 2.2. Let μ and ν be two fuzzy numbers represented parametrically by

$$\{(a(\alpha), b(\alpha), \alpha) : \alpha \in I\} \quad \text{and} \quad \{(c(\alpha), d(\alpha), \alpha) : \alpha \in I\},$$

respectively. We say that $\mu \preceq \nu$ if

$$a(\alpha) \leq c(\alpha) \quad \text{and} \quad b(\alpha) \leq d(\alpha) \text{ for each } \alpha \in I.$$

We call $\preceq$ the fuzz-max order on $\mathcal{F}$.